Electrodynamic Spacetime Relativity: Extending Special Relativity and Maxwell's Equations based on the Electric Potential Limit Constant[1]

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Abstract: This paper introduces the electric potential limit constant as a hypothesized new fundamental physical constant and explores its consequences for spacetime structure and classical electrodynamics. Based on the intrinsic symmetry between this constant and the intrinsic speed of light, the spacetime framework of Special Relativity is extended from the real domain to the complex and biquaternion domains, leading to a nine-dimensional spacetime framework termed Electrodynamic Spacetime Relativity (ESR). Within this formulation, standard Special Relativity and Electric Potential Relativity naturally emerge as two limiting sectors of a more general structure.

On this basis, a nonlinear electromagnetic framework compatible with Electric Potential Relativity is developed. Within the assumptions of the present model, the theory is formulated to preserve generalized gauge invariance while retaining the absolute invariance of electric charge and the fine-structure constant. The resulting modified Coulomb's law predicts a nonclassical short-distance behavior of the effective interaction. For the idealized point-charge model considered here, the corresponding electric field self-energy is naturally regularized to a finite value. This provides a rigorous geometric resolution to this long-standing fundamental difficulty in classical electrodynamics.

The framework also predicts macroscopic electric-potential-induced time dilation, redshift, and lensing-like effects, suggesting concrete experimental tests, including the concepts of an electric-potential telescope and an electric-potential Michelson interferometer. Possible implications for further extensions toward gravitation and quantum-scale physics are briefly discussed.

Keywords: Electrodynamic Spacetime Relativity; Electric Potential Relativity; electric potential limit constant; nonlinear Maxwell equations; idealized point-charge model; generalized gauge invariance.

1. Introduction

Maxwell's equations [1] and Special Relativity [2] together constitute one of the central foundations of modern theoretical physics. Their mutual consistency has profoundly shaped the modern understanding of fields, spacetime, and electromagnetic interactions. However, within classical electrodynamics, the idealized point-charge model gives rise to a long-standing difficulty: the electric potential and electric field diverge at the charge center, and the

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[1] **Version and Methodology Note:** The foundational framework of this theory was initially established by the author in 2011, and the complete nine-dimensional spacetime architecture was gradually developed and matured in subsequent versions after 2012. As the latest revised version, this paper systematically reconstructs the theory: 1) To enhance mathematical transparency, a novel method for directly deriving the biquaternion Lorentz transformation from the classical vector Lorentz transformation is adopted; 2) To ensure content continuity, two previous papers on Electrodynamic Spacetime Relativity and nonlinear electromagnetic theory are merged; 3) Concurrently, the conventions of standard classical literature (e.g., Maxwell, Einstein, Dirac) are strictly followed to ensure rigorous theoretical consistency.

[2] **Author Biography:** The scientific concept of the "electric potential limit constant" originated during the author's studies at Chongqing University in China. From 1982 to 1998, the author worked at the Xi'an Research Institute of China Coal Technology & Engineering Group, engaging in engineering physics experiments and theoretical research, and was promoted to Senior Engineer. After relocating to Canada in 1998, the author utilized spare time to develop this deeply held concept into the systematic Electrodynamic Spacetime Relativity, aiming to inherit and expand upon Special Relativity and electromagnetic theory, and to fundamentally resolve the point charge divergence problem in classical electrodynamics. Currently residing in Ontario, Canada (retired). Email: yingtaoyang.physics@gmail.com

corresponding electromagnetic field self-energy becomes divergent [3, 15]. This self-energy problem remains one of the most persistent conceptual difficulties in the classical description of charged particles.

Quantum Electrodynamics (QED) addresses these divergent terms through renormalization procedures, achieving extremely accurate agreement with experiment. Nevertheless, renormalization does not by itself provide a geometric or spacetime-level explanation for the origin of these divergences in the idealized point-charge model. As emphasized by Dirac [4], the subtraction of infinities, although operationally successful, leaves open the question of whether a deeper structural reformulation may exist. This motivates a rigorous re-examination of electromagnetic theory at a more fundamental theoretical level.

In this paper, the electric potential limit constant, denoted by Φ_0, is introduced as a hypothesized new fundamental physical constant. The central idea is that electric potential, much like velocity in Special Relativity, may possess an intrinsic upper bound. The purpose of this hypothesis is not to claim an experimentally established constant at the outset, but to thoroughly investigate the theoretical consequences of assuming such a bound for spacetime structure and classical electrodynamics. Within the idealized point-charge model considered here, this assumption logically leads to a finite upper bound for the potential, thereby yielding a finite electromagnetic field self-energy.

Based on this hypothesis, the spacetime framework of Special Relativity is extended from the real domain to the complex and biquaternion domains, leading to a nine-dimensional spacetime framework termed Electrodynamic Spacetime Relativity (ESR). Within this formulation, velocity-based relativity (Special Relativity) and electric-potential-based relativity (Electric Potential Relativity) naturally emerge as two limiting sectors of a broader unified structure. On this basis, a nonlinear electromagnetic framework compatible with Electric Potential Relativity is developed. In contrast to the Born–Infeld theory [5] and the Heisenberg–Euler theory [6], which introduce nonlinearity phenomenologically through modified electromagnetic Lagrangians, the present construction is strictly motivated by the covariance requirements of the proposed high-dimensional spacetime framework.

Within the assumptions of the present model, the resulting theory is formulated to preserve generalized gauge invariance while retaining the absolute invariance of electric charge and the fine-structure constant. The modified Coulomb's law exhibits a nonclassical short-distance behavior of the effective interaction; consequently, for the idealized point-charge model considered here, the electromagnetic field self-energy is naturally regularized to a finite value. The framework also predicts macroscopic electric-potential-induced time dilation, redshift, and lensing-like effects, suggesting concrete experimental tests. Possible implications for further extensions toward gravitation and quantum-scale physics are briefly discussed, though these broader issues remain beyond the main scope of the present paper.

The remainder of this paper is organized as follows. Section 2 introduces the hypothesis of the electric potential limit constant and develops the Electrodynamic Spacetime Relativity framework. Section 3 derives the corresponding nonlinear Maxwell's equations and discusses their main physical consequences. Section 4 presents concepts for possible experimental verification and astronomical observation. Section 5 summarizes the main results, and finally, Section 6 outlines directions for further investigation.

The main abbreviations and symbols used in this paper are as follows:

C_0: Intrinsic Speed of Light

Φ_0: Electric Potential Limit Constant

ESR: Electrodynamic Spacetime Relativity

SR: Special Relativity

PR: Electric Potential Relativity

CESR: Complex Electrodynamic Spacetime Relativity

2. Electrodynamic Spacetime Relativity

Electrodynamic Spacetime Relativity (ESR) constitutes the theoretical foundation of this paper. By introducing a new fundamental physical constant—the electric potential limit constant (Φ_0)—and building upon Special Relativity, this theory extends the spacetime descriptive paradigm from the real number domain to the complex and biquaternion domains. Consequently, a high-dimensional spacetime framework that achieves the unification of "velocity relativity" and "electric potential relativity" is constructed. Just as Special Relativity establishes the inherent speed of light (C_0) as the physical boundary in the kinematic domain, Electrodynamic Spacetime Relativity further reveals that an intrinsic limit constant Φ_0 equally exists in the electric potential domain. Physically, this discovery establishes a profound dual symmetry between "velocity and electric potential"; mathematically, it corresponds to the real-imaginary symmetry of the "complex coefficients" in biquaternions. This provides the possibility of unifying classical electrodynamics and spacetime geometry within a higher-dimensional dual structure.

2.1 Hypothesis of the Electric Potential Limit Constant

In classical electrodynamics, the electric potential obeys the linear superposition principle. Accordingly, in the ideal point-charge model, the potential is not subject to an intrinsic upper bound within the classical theory. According to electromagnetic theory, the equations for calculating the electric potential ϕ, electric field intensity $\boldsymbol{E}$, and electric field energy density w_{e} of an ideal stationary point charge q are as follows:

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \tag{1}$$

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}} \tag{2}$$

$$w_{\mathrm{e}} = \frac{1}{2}\varepsilon_0 E^2 \tag{3}$$

The above three equations indicate that the electric potential, electric field intensity, and energy density at the center of the point charge are all infinite. Therefore, the total energy E_{q} [3] obtained by integrating the electric field energy of this point charge over the entire spatial volume $\mathbb{R}^3$ is:

$$E_{\mathrm{q}} = \iiint_{\mathbb{R}^3} \frac{1}{2}\varepsilon_0 E^2 \mathrm{d}v = \int_0^{\infty} \frac{q^2}{8\pi\varepsilon_0 r^2}\mathrm{d}r = \lim_{r\to 0}\frac{q^2}{8\pi\varepsilon_0 r} = +\infty \tag{4}$$

where the volume differential is $\mathrm{d}v = 4\pi r^2 \mathrm{d}r$, *and* ε_0 is the vacuum permittivity.

Consequently, the total electric field energy associated with a point charge tends to infinity, representing a fundamental challenge that cannot be resolved within the framework of classical electrodynamics.

If the elementary point charge is not a point but rather a charge distributed within a localized spatial region, a series of new problems emerge, such as violating the quantized nature of the elementary charge [7], and the repulsive forces between charge fractions destroying the stability of the charge [8, Section 16.2], among others. Thus, the majority of modern physical theories, including quantum field theory, continue to use the point-charge model.

However, the divergence problem of the electric field energy of a point charge remains fundamentally unresolved. Although renormalization methods have achieved computational success, their physical significance is limited to "eliminating infinities" and does not reveal the fundamental cause of energy divergence. As explicitly pointed out by Dirac, renormalization is merely a mathematical patch rather than a physical solution; the true answer should originate from a deeper fundamental theoretical framework. Although the Born-Infeld [5] and Heisenberg-Euler [6] theories historically attempted to mitigate the field energy divergence problem via non-linear terms, they both rely on empirical constructions, lacking a logical derivation originating from spacetime structure or fundamental symmetries. Therefore, despite possessing some physical heuristic significance, these theories fail to reveal the essential cause of the divergence in point-charge energy. Is it possible, starting from first principles, to modify electrodynamics and thoroughly overcome the infinite divergence of electric field energy while preserving the point charge model? Based on in-depth analysis, this paper proposes a critical hypothesis: a finite limit constant for the electric potential may exist, denoted as Φ_0. This concept originates from an analogy to the "velocity limit" in Special Relativity. Since an upper limit for velocity (C_0) exists in the kinematic domain, a finite upper limit constant (Φ_0) may also exist in the electric potential domain. If this limit indeed exists, the electric potential at the center of a point charge will no longer approach infinity, but will instead be a finite constant Φ_0. Under this condition, the electric field intensity and electric field energy density at the center are no longer infinite, but tend toward zero. Consequently, the total electric field energy of the point charge becomes a finite value; this hypothesis provides a completely new theoretical pathway to thoroughly resolve the point charge energy divergence problem and clears the theoretical obstacles for the further development of the point particle model. Elevating the electric potential limit constant Φ_0 to a fundamental physical constant similar to the speed of light C_0 implies that the theoretical structure of the electromagnetic field will undergo a fundamental change. Due to the strict covariant relationship between Maxwell's equations and Special Relativity, the hypothesis of introducing Φ_0 not only necessitates modifications to the electromagnetic equations but also requires the spacetime theory itself to be correspondingly extended. Within this framework, electric potential and velocity exhibit a high degree of similarity in their physical characteristics:

- Both possess their respective limit constants (C_0 and Φ_0);
- Both obey nonlinear superposition laws;
- Both must satisfy the principle of relativity and the requirements of covariance.

This indicates that a deep symmetry may exist between electric potential and velocity. By inheriting the formal structure of Special Relativity and extending its "principle of the constancy of the speed of light" and "principle of relativity," a more generalized spacetime theory—Electrodynamic Spacetime Relativity—can thus be established.

2.2 Complex Electrodynamic Spacetime Relativity

To elucidate the coupling mechanism between velocity and electric potential, this section first derives the fundamental transformation relations of three-dimensional spacetime (i.e., a two-dimensional complex space plus one-dimensional time) within a complex plane space constituted by one-dimensional velocity (real space) and one-dimensional electric potential (imaginary space). A more general nine-dimensional spacetime formalism (eight-dimensional biquaternion space plus one-dimensional time) will be presented in Section 2.3.

2.2.1 Review of Fundamental Postulates of Special Relativity

It is well known that Einstein's Special Relativity [2] is based on two postulates regarding inertial reference frames:

• Postulate (A1) Principle of Relativity for Inertial Frames: The laws of physics have the same form in all inertial reference frames. Here, an "inertial reference frame" refers to an ideal reference frame moving in uniform rectilinear motion and free from external forces.

• Postulate (A2) Principle of the Constancy of the Speed of Light: In a vacuum, the speed of light C_0 has the same value in all inertial reference frames, independent of the state of motion of the light source or the observer.

From postulates (A1) and (A2), the standard Lorentz transformation can be derived:

$$X' = \gamma(V_x)(X - V_x\, t) \tag{5}$$

$$Y' = Y, \quad Z' = Z \tag{6}$$

$$t' = \gamma(V_x)\left(t - \frac{V_x}{C_0{}^2}X\right) \tag{7}$$

$$\gamma(V_x) = \frac{1}{\sqrt{1-\left(\frac{V_x}{C_0}\right)^2}} \tag{8}$$

Here, the Lorentz factor in one-dimensional Special Relativity is explicitly denoted as a function $\gamma(V_x)$, and it is referred to as the velocity Lorentz factor. This notation is intended to pave the way for subsequent theoretical extensions in this paper: rather than treating γ merely as a kinematic coefficient, it is redefined as a Generalized Lorentz Factor $\gamma(\text{reference frame variable})$ that characterizes the "nonlinear geometric effects generated when a physical quantity approaches its limit." In the subsequent Electrodynamic Spacetime Relativity (ESR), this mathematical notation will be utilized to introduce the electric potential Lorentz factor $\gamma(\phi)$, with the reference frame electric potential ϕ as the independent variable. This aims not only to reveal the profound physical isomorphism between the velocity limit and the electric potential limit but also lays the mathematical and physical foundation for the synchronous generalization of the Lorentz factor as Special Relativity is extended into high-dimensional Electrodynamic Spacetime Relativity.

In the Lorentz transformation, (X', Y', Z', t') are the spacetime coordinates of an event in the inertial frame Σ_r', and (X, Y, Z, t) are the spacetime coordinates of the same event in the reference frame Σ_r. The inertial reference frame Σ_r' moves uniformly at a velocity V_x relative to the inertial reference frame Σ_r along the $+X$ direction. The above transformation is equivalent to performing a Lorentz transformation on the (X, tC_0) plane, while the Y', Z' components remain invariant.

2.2.2 Fundamental Postulates of Electric Potential Relativity

In addition to inertial motion reference frames, it is possible to conceive of another type of reference frame—the equipotential reference frame. According to Coulomb's law and Gauss's law, the electric potential is equal everywhere inside a closed conductor, and the electric field is zero. Therefore, the principle of relativity holds equally valid within an equipotential reference frame. If an electric potential limit constant Φ_0 exists, which is symmetrically positioned with respect to the speed of light C_0 in physical status, this paper proposes:

• Postulate (A3) Principle of Relativity for Electric Potential: The laws of physics have the same form in all equipotential reference frames. An equipotential reference frame is an ideal reference frame in which the electric potential is constant at all points in space, and the electric field is zero, such as the interior of a closed-conductor cavity.

• Postulate (A4) Principle of the Invariance of the Electric Potential Limit: An electric potential limit constant Φ_0 exists. In all equipotential stationary reference frames in a vacuum, Φ_0 takes the same value, and it does not change with the choice of reference frame.

Postulates (A3) and (A4) are structurally symmetric to (A1) and (A2), implying that an "Electric Potential Relativity" parallel to Special Relativity can be established. Therefore, it is necessary to extend the concepts of relevant fundamental physical quantities and establish the correspondence between them.

2.2.3 Electric Potential and the Imaginary Velocity State

To unify electric potential with inertial reference frames, this paper introduces a new physical concept—the Imaginary velocity state. This concept converts electric potential into an imaginary velocity iV_ϕ via a proportionality constant, thereby establishing a symmetric relationship between electric potential and velocity. To distinguish it from ordinary velocity, it is termed the imaginary velocity state, and let V_ϕ be the imaginary velocity state coefficient. Based on the symmetry between potential and velocity, this yields:

$$iV_\phi = \psi\phi \tag{9}$$

Here, ψ is the conversion coefficient, and the imaginary unit is $i = \sqrt{-1}$.

When the electric potential takes its limit value $\phi = \Phi_0$, the imaginary velocity state should approach its limit $i\mathrm{C}_0$; therefore, the correspondence between the electric potential limit constant Φ_0 and the speed of light C_0 is:

$$iC_0 = \psi\Phi_0 \tag{10}$$

From this, it follows that:

$$\psi = \frac{C_0}{\Phi_0} i \tag{11}$$

This indicates that the electric potential limit constant Φ_0 corresponds to the speed of light C_0, and an equipotential reference frame can be regarded as an "imaginary velocity state inertial frame."

The introduction of the electric potential limit constant Φ_0 signifies that the description of electrodynamics will transition from classical global linear superposition to a bounded geometric structure. This logical evolution shares a profound physical equivalence with Special Relativity's introduction of the speed of light C_0 to replace Galilean transformations. From this perspective, the reference frame potential state ϕ is viewed as an intrinsic parameter marking the physical ground state, rather than an arbitrarily translatable potential function field as in traditional gauges.

Crucially, the classical U(1) gauge symmetry encompassing scalar and vector potentials remains strictly conserved within each local tangent space, which will be discussed in depth in Section 3.3, "Generalized Gauge Invariance and Conservation Laws." Nonlinear effects are solely manifested in the transformation logic between tangent spaces of different potential states. While maintaining local gauge consistency, this geometric reconstruction naturally eliminates the singularity dilemma of the point-charge self-energy divergence by renormalizing the spatial scale, thereby predicting a series of new physical effects beyond the classical paradigm.

2.2.4 Definition of Complex Position State, Complex Velocity State, and Complex Potential State

To investigate more universal spacetime correlations, this research extends the motion representation and the spacetime descriptive paradigm from the real-number domain of Special Relativity to the complex domain. Within this framework, a more general motion can be abstracted as a "motion state" on a complex plane. If the motion representation of a reference frame simultaneously contains a real-part motion component (inertial motion state) and an imaginary-part motion component (equipotential motion state), it is termed a complex electrodynamic inertial reference frame. The theory describing the spacetime transformation logic between such reference frames is Complex Electrodynamic Spacetime Relativity.

Suppose there exists a stationary frame Σ_c and a moving frame Σ'_c on the complex plane, with their corresponding real and imaginary axes parallel to each other. Due to the unordered nature of complex numbers, it is stipulated that: physically measurable quantities correspond to the "modulus" of complex numbers, while spacetime geometry is described by "states." The complex position states of event P in the two frames are R_c and R'_c, respectively; the components of R_c and R'_c are (F,X) and (F',X'); their corresponding physical distances are given by the moduli $|R_\mathrm{c}|$ and $|R'_\mathrm{c}|$. Let t and t' be the time in reference frames Σ_c and Σ'_c, respectively. Suppose Σ'_c moves relative to Σ_c with a complex velocity state V_c, its components and modulus being (V_ϕ,V_x) and $|V_\mathrm{c}|$, respectively. The relevant physical quantities are defined as follows:

$$R_\mathrm{c} = X + iF \tag{12}$$

$$|R_\mathrm{c}| = \sqrt{X^2 + F^2} \tag{13}$$

$$R'_\mathrm{c} = X' + iF' \tag{14}$$

$$|R'_{\mathrm{c}}| = \sqrt{X'^2 + F'^2} \tag{15}$$

$$V_{\mathrm{c}} = V_x + iV_\phi = V_x + i\frac{C_0\phi}{\Phi_0} \tag{16}$$

$$|V_{\mathrm{c}}| = \sqrt{{V_x}^2 + {V_\phi}^2} = \sqrt{{V_x}^2 + \left(\frac{C_0\phi}{\Phi_0}\right)^2} \tag{17}$$

$$\text{Where，}\ V_\phi = \frac{C_0}{\Phi_0}\phi \tag{18}$$

Because real velocity corresponds to imaginary potential, and imaginary velocity corresponds to real-domain potential, the complex potential state ϕ_{c} and its modulus $|\phi_{\mathrm{c}}|$ corresponding to the complex velocity state $V_{\mathrm{c}} = V_x + iV_\phi$ are defined as:

$$\phi_{\mathrm{c}} = \phi + i\phi_x \tag{19}$$

$$|\phi_{\mathrm{c}}| = \sqrt{{\phi_x}^2 + \phi^2},\ |\phi_x| = \frac{V_x\Phi_0}{C_0} \tag{20}$$

$$V_{\mathrm{c}} = \psi\phi_{\mathrm{c}},\ \psi = i\frac{C_0}{\Phi_0},\ (i^2 = -1) \tag{21}$$

$$V_{\mathrm{c}} = V_x + iV_\phi = i\frac{C_0}{\Phi_0}(\phi + i\phi_x) = \frac{C_0}{\Phi_0}(-\phi_x + i\phi) \tag{22}$$

$$V_x = -\frac{C_0}{\Phi_0}\phi_x$$

Through the transformation factor ψ, a spacetime duality is revealed: the real velocity V_x in physical space maps to the imaginary component ϕ_x in potential space, while the physical real potential ϕ maps to the imaginary component V_ϕ in velocity space.

On this basis, a real-valued scalar operation specific to Electrodynamic Spacetime Relativity—the "electrodynamic real scalar product" (denoted by the symbol ·)—is defined. Depending on the manifestation of the dynamical state, this real scalar product is strictly defined as the projection of the underlying Hermitian inner product onto different phase axes:

When the state parameter is the velocity state V_{c}, the physical attributes map to the real axis, and its real scalar product is defined as the real part of the Hermitian inner product:

$$R_{\mathrm{c}} \cdot V_{\mathrm{c}} := \mathrm{Re}\langle R_{\mathrm{c}}, V_{\mathrm{c}}\rangle = \mathrm{Re}\left(R_{\mathrm{c}}^\dagger V_{\mathrm{c}}\right)$$

When the state parameter is the potential state ϕ_{c}, the physical attributes map to the imaginary axis, and its real scalar product is defined as the negative imaginary part of the Hermitian inner product:

$$R_{\mathrm{c}} \cdot \phi_{\mathrm{c}} := -\mathrm{Im}\langle R_{\mathrm{c}}, \phi_{\mathrm{c}}\rangle = -\mathrm{Im}\left(R_{\mathrm{c}}^\dagger \phi_{\mathrm{c}}\right)$$

From this, it follows that:

$$R_{\mathrm{c}} \cdot V_{\mathrm{c}} = \mathrm{Re}[(X - iF)(V_x + iV_\phi)] = XV_x + FV_\phi = \frac{C_0}{\Phi_0}(-X\phi_x + F\phi) \tag{23}$$

$$R_{\mathrm{c}} \cdot \phi_{\mathrm{c}} = -\mathrm{Im}[(X - iF)(\phi + i\phi_x)] = -X\phi_x + F\phi \tag{24}$$

$$R_{\mathrm{c}} \cdot V_{\mathrm{c}} = \frac{C_0}{\Phi_0} R_{\mathrm{c}} \cdot \phi_{\mathrm{c}} \tag{25}$$

2.2.5 Fundamental Postulates of Complex Electrodynamic Spacetime Relativity

The preceding discussion demonstrates that the complex velocity V_{c} and the complex potential ϕ_{c} can mutually correspond through a linear relationship; thus, a complex potential reference frame is also a type of complex electrodynamic inertial reference frame. Consequently, the four postulates (A1) to (A4) of Special Relativity and Electric Potential Relativity can be unified and formulated as the following two postulates within complex electrodynamic spacetime.

• Postulate (A5) Principle of Relativity for Complex Electrodynamic Spacetime: The laws of physics have the same form in any complex electrodynamic inertial reference frame.

Here, a complex electrodynamic inertial reference frame refers to a reference frame whose state of motion can be described by the complex velocity state $V_{\mathrm{c}} = V_x + iV_\phi$, where the real part corresponds to the inertial motion state and the imaginary part corresponds to the potential state, or equivalently, by the complex potential state $\phi_c = \phi + i\phi_x$.

• Postulate (A6) Principle of the Invariance of the Speed of Light / Electric Potential Limit in Complex Electrodynamic Spacetime: In any complex electrodynamic inertial reference frame, the modulus of the complex velocity state of light is an invariant limit constant C_0 (i.e., the inherent speed of light in classical vacuum), which does not change with the choice of reference frame. Equivalently, the limit of the modulus of the complex potential state ϕ_c is an invariant constant Φ_0, which also does not change with the choice of reference frame.

From postulate (A6), it can be inferred that: in a reference frame with a non-zero electric potential, the coordinate speed of light C_0', which is the projection of light velocity into physical real space, is less than C_0; however, in complex space, the modulus of the complex velocity state of light remains equal to the inherent speed of light C_0. The generalized principle of the constancy of the speed of light still strictly holds, namely:

$$C_0{}^2 = C_0'^2 + V_\phi{}^2 = C_0'^2 + \left(\frac{C_0}{\Phi_0}\phi\right)^2 \tag{26}$$

When $V_x = 0$ and $V_c = V_\phi i$, the above two postulates (A5) and (A6) degenerate into postulates (A3) and (A4) of Electric Potential Relativity.

When $V_\phi = 0$ and $V_c = V_x$, the above two postulates (A5) and (A6) degenerate into postulates (A1) and (A2) of Special Relativity.

Therefore, Special Relativity and Electric Potential Relativity are two special cases of Complex Electrodynamic Spacetime Relativity that are formally symmetric and physically complementary.

2.2.6 General Complex Relativity

According to the vector equation for the Lorentz transformation in an arbitrary direction in three-dimensional space in Reference [8], by simplifying through dimensionality reduction, the vector equation for the Lorentz transformation in an arbitrary direction in two-dimensional space can be obtained. This enables the planar two-dimensional vector equation to be further transformed into a complex Lorentz transformation equation:

Suppose the coordinate axes of two inertial reference frames Σ and Σ' remain parallel, and their two-dimensional position vectors are respectively: $\boldsymbol{r}' = \boldsymbol{x}' + \boldsymbol{y}'$ and $\boldsymbol{r} = \boldsymbol{x} + \boldsymbol{y}$. The velocity vector of the reference frame Σ' relative to Σ is: $\boldsymbol{v} = \boldsymbol{v}_x + \boldsymbol{v}_y$, and $|\boldsymbol{v}|^2 = v_x^2 + v_y^2$. Assuming the direction of $\boldsymbol{v}$ can be any direction within the two-dimensional plane, the spacetime transformation can be written as the following compact vector equations:

$$\boldsymbol{r}' = \boldsymbol{r} + (\gamma(\boldsymbol{v}) - 1)\frac{(\boldsymbol{v}\cdot\boldsymbol{r})\boldsymbol{v}}{|\boldsymbol{v}|^2} - \gamma(\boldsymbol{v})\boldsymbol{v}t \tag{27}$$

$$t' = \gamma(\boldsymbol{v})\left(t - \frac{\boldsymbol{v}\cdot\boldsymbol{r}}{C_0{}^2}\right) \tag{28}$$

Where, $\gamma(\boldsymbol{v}) = \frac{1}{\sqrt{1-(|\boldsymbol{v}|/C_0)^2}}$, and the dot product of the two-dimensional vectors is: $\boldsymbol{v} \cdot \boldsymbol{r} = v_x x + v_y y$.

According to Eq. (27), it can be decomposed into two vector equations along the x-axis and y-axis:

$$\boldsymbol{x}' = \boldsymbol{x} + (\gamma(\boldsymbol{v}) - 1)\frac{(\boldsymbol{v}\cdot\boldsymbol{r})\boldsymbol{v}_x}{|\boldsymbol{v}|^2} - \gamma(\boldsymbol{v})\boldsymbol{v}_x t \tag{29}$$

$$\boldsymbol{y}' = \boldsymbol{y} + (\gamma(\boldsymbol{v}) - 1)\frac{(\boldsymbol{v}\cdot\boldsymbol{r})\boldsymbol{v}_y}{|\boldsymbol{v}|^2} - \gamma(\boldsymbol{v})\boldsymbol{v}_y t \tag{30}$$

Where, $\gamma(\boldsymbol{v}) = \frac{1}{\sqrt{1-(|\boldsymbol{v}|/C_0)^2}}$

Further removing the unit vectors of the x-axis and y-axis respectively, and letting the dot product be $\boldsymbol{v} \cdot \boldsymbol{r} = v_x x + v_y y = d$, $|v|^2 = v_x^2 + v_y^2$. Two scalar equations can be obtained:

$$x' = x + (\gamma(v) - 1)\frac{v_x d}{|v|^2} - \gamma(v)v_x t \tag{31}$$

$$y' = y + (\gamma(v) - 1)\frac{v_y d}{|v|^2} - \gamma(v)v_y t \tag{32}$$

$$t' = \gamma(v)\left(t - \frac{d}{{C_0}^2}\right) \tag{33}$$

Where, $\gamma(v) = \frac{1}{\sqrt{1-(|v|/C_0)^2}}$

Now, the concept of complex numbers is introduced: by treating the above two-dimensional real plane space as a complex plane, the x-axis and x′-axis are kept as real axes, meaning Eq. (31) remains unchanged; the y-axis and y′-axis are transformed into imaginary axes. Therefore, by multiplying both sides of Eq. (32) by the imaginary unit i, where $i^2 = -1$, the "imaginary-transformed" equation is obtained:

$$iy' = iy + i(\gamma(v) - 1)\frac{v_y d}{|v|^2} - i\gamma(v)v_y t \tag{34}$$

Let: $x' + iy' = z'$ (complex coordinate), $x + iy = z$ (complex coordinate), $v_x + iv_y = v$ (complex velocity scalar), $|v|^2 = v_x^2 + v_y^2$. Note that $d = \boldsymbol{v} \cdot \boldsymbol{r} = v_x x + v_y y$ is equivalent to the real part of the Hermitian inner product of complex numbers, $\mathrm{Re}\langle z^\dagger v\rangle$, in the complex plane. Herein, the real scalar product, i.e., the dot product, of complex numbers is defined as:

$$z \cdot v = \mathrm{Re}\langle z^\dagger v\rangle = \mathrm{Re}\langle(x - iy)(v_x + iv_y)\rangle = xv_x + yv_y \tag{35}$$

$$d = \boldsymbol{v} \cdot \boldsymbol{r} = z \cdot v = \mathrm{Re}\langle z^\dagger v\rangle \tag{36}$$

Substituting d in Eqs. (31),(33),(34) with $z \cdot v$, and combining Eqs. (31) and (34) by addition into a single complex equation yields the general complex relativity equation:

$$z' = z + (\gamma(v) - 1)\frac{v(z \cdot v)}{|v|^2} - \gamma(v)vt \tag{37}$$

$$t' = \gamma(v)\left(t - \frac{z \cdot v}{{C_0}^2}\right) \tag{38}$$

Where, $\gamma(v) = \frac{1}{\sqrt{1-(|v|/C_0)^2}}$

2.2.7 Complex Electrodynamic Spacetime Relativity and Electric Potential Relativity

The aforementioned general complex relativity is a spacetime relativity in three dimensions; it can serve as the spacetime relativity for an arbitrary complex plane space plus time. Therefore, by substituting the corresponding complex physical quantities ${R'}_{\mathrm{c}}$, R_{c}, and V_{c} into the general complex relativity Eqs. (37) and (38) respectively, Complex Electrodynamic Spacetime Relativity for a reference frame in a complex velocity state can be obtained:

$$R'_{\mathrm{c}} = R_{\mathrm{c}} + (\gamma(V_{\mathrm{c}}) - 1)\frac{V_{\mathrm{c}}(R_{\mathrm{c}} \cdot V_{\mathrm{c}})}{|V_{\mathrm{c}}|^2} - \gamma(V_{\mathrm{c}})V_{\mathrm{c}}t \tag{39}$$

$$t' = \gamma(V_{\mathrm{c}})\left(t - \frac{R_{\mathrm{c}} \cdot V_{\mathrm{c}}}{{C_0}^2}\right) \tag{40}$$

Where, $\gamma(V_{\mathrm{c}}) = \frac{1}{\sqrt{1-(|V_{\mathrm{c}}|/C_0)^2}}$

This describes a spacetime relativity where a reference frame possesses both an inertial velocity and an equipotential state.

A complex velocity state can be converted into a complex potential state. Therefore, substituting Eqs. (21) and (25) into Eqs. (39) and (40) yield Complex Electrodynamic Spacetime Relativity for a reference frame in a complex potential state:

$$R'_{\mathrm{c}} = R_{\mathrm{c}} + i(\gamma(\phi_{\mathrm{c}}) - 1)\frac{\phi_{\mathrm{c}}(R_{\mathrm{c}} \cdot \phi_{\mathrm{c}})}{|\phi_{\mathrm{c}}|^2} - i\gamma(\phi_{\mathrm{c}})\frac{C_0}{\Phi_0}\phi_{\mathrm{c}}t \tag{41}$$

$$t' = \gamma(\phi_{\mathrm{c}})\left(t - \frac{R_{\mathrm{c}} \cdot \phi_{\mathrm{c}}}{\Phi_0 C_0}\right) \tag{42}$$

Where, $\gamma(\phi_{\mathrm{c}}) = \frac{1}{\sqrt{1-(|\phi_{\mathrm{c}}|/\Phi_0)^2}}$

When $V_\phi = 0$, this theory reduces to standard Special Relativity, as shown in Eqs. (5)-

(8).

When $V_x = 0$, the electrodynamic inertial reference frame degenerates into an intrinsic equipotential reference frame, meaning that within this reference frame, an observer only experiences a pure electric potential ϕ, without spatial velocity ($V_x = 0$). This yields Electric Potential Relativity:

$$iF' = i\gamma(\phi)\left(F - \frac{C_0}{\Phi_0}\phi t\right) \tag{43}$$

$$X' = X \tag{44}$$

$$t' = \gamma(\phi)\left(t - \frac{F\phi}{\Phi_0 C_0}\right) \tag{45}$$

Where, $\gamma(\phi) = \frac{1}{\sqrt{1-(\phi/\Phi_0)^2}}$

Electric Potential Relativity is formally perfectly symmetric with Special Relativity. Eq. (43) indicates that the coordinate iF' in imaginary space (i.e., the electric potential dimension) undergoes a Lorentz transformation, meaning that in a reference frame with a potential difference, not only does time dilate, but the length modulus in imaginary space (i.e., the potential interval) also experiences a relativistic contraction effect. This phenomenon forms a perfect symmetric complementarity with Special Relativity: in Special Relativity, velocity induces a contraction of length in real space; whereas in Electric Potential Relativity, electric potential induces a contraction of length in imaginary space.

Eq. (44) reveals an important physical characteristic: unlike the spatial contraction caused by velocity in Special Relativity, a pure potential difference does not induce geometric deformation of real physical space. This unique property is designated herein as "Potential Spatial Invariance". This property ensures the form invariance of the spatial differential operator (∇) under equipotential transformations, serving as the crucial geometric foundation for the subsequent derivation of form-covariant nonlinear Maxwell's equations.

From Eq. (45), it is known that when F is invariant ($\Delta F = 0$), a functional relationship exists between the proper time $\Delta \mathrm{t}$ and the coordinate time $\Delta \mathrm{t}'$ that is dependent on the reference frame's electric potential. That is, when $V_x = 0$, a proper time dilation effect purely induced by the electric potential is generated:

$$\Delta t' = \frac{1}{\sqrt{1-(\phi/\Phi_0)^2}}\Delta t \tag{46}$$

Performing a Taylor expansion on the above equation yields:

$$\Delta t' \approx \Delta t\left(1 + \frac{1}{2}\frac{\phi^2}{{\Phi_0}^2}\right) \tag{47}$$

It is thus evident that because the value of Φ_0 is extremely large, in conventional electromagnetic environments, $(\phi/\Phi_0)^2$ approaches zero, rendering the corrective effect of the electric potential on time a second-order infinitesimal quantity that is extremely difficult to observe. Consequently, this effect is unobservable in everyday (low-potential) environments; it only becomes significant under extremely high-potential conditions (such as astrophysical environments or the cores of microscopic particles).

2.3 Biquaternion Electrodynamic Spacetime Relativity

The complex relativity in the preceding section merely revealed the intrinsic relationship between spacetime and electric potential on a simplified two-dimensional complex plane. To accurately describe the real three-dimensional physical world, the one-dimensional scalar dimension of real numbers must be expanded into three-dimensional vector dimensions, and its algebraic structure is naturally generalized from complex numbers to biquaternions (quaternions with complex coefficients). Based on mathematical completeness and physical symmetry, a complete biquaternion contains eight independent components. Therefore, the mathematical essence of the constructed theoretical model is the deep integration of an eight-dimensional biquaternion electrodynamic space and one-dimensional time.

The so-called "Biquaternion Electrodynamic Space" physically unifies the degrees of freedom of motion and electric potential in space. In mathematical structure, it consists of two mutually conjugate four-dimensional subspaces. Based on the principle of relativity, the mapping of the parameters describing the reference frame state to the real and imaginary parts depends entirely on the observer's choice of reference frame: for a standard velocity inertial observer (i.e., uniformly adopting a biquaternion velocity to describe the physical state of the reference frame), the physical motion velocity manifests as the real velocity, while the electric potential is transformed into an imaginary velocity state.

Conversely, for an equipotential observer (i.e., uniformly adopting a biquaternion electric potential to describe the physical state of the reference frame), the electric potential manifests as the real potential, while the physical motion velocity is transformed into an imaginary potential state. This characteristic of "real-imaginary interchange" of the state parameters profoundly reveals the underlying geometric duality between velocity and electric potential. This means that biquaternion velocity and biquaternion electric potential are completely equivalent in mathematical structure and strictly dual in physical status. The specific choice of which parameter to use to describe the physical state of the system essentially depends on the physical characteristics and observation scale of the object under study. It must be emphasized that the imaginary dimensions herein correspond physically to the internal degrees of freedom of the system, rather than newly added macroscopic spatial dimensions.

This geometric approach is highly consistent in physical thought with the fifth dimension in Kaluza-Klein theory [13] or the internal phase space in gauge field theory. To maintain conciseness of expression while balancing mathematical rigor, this holistic extended structure, integrating eight-dimensional spatial degrees of freedom and one-dimensional time, is formally named "Nine-dimensional Electrodynamic Spacetime," hereafter referred to as "nine-dimensional spacetime."

2.3.1 From Hamilton Quaternions to Biquaternions

Mathematically, classical quaternions are defined by real numbers. Herein, they are referred to as Hamilton Quaternions or real quaternions, denoted as H, which correspond to the sum of classical physical vectors and scalars.

$$H = r_1 + r_2\mathbf{i} + r_3\mathbf{j} + r_4\mathbf{k} \tag{48}$$

A Hamilton quaternion is denoted as:

$$\sum_{\mu=1}^{4} r_\mu\, e_\mu \tag{49}$$

Where $e_1 = 1$, $e_2 = \mathbf{i}$, $e_3 = \mathbf{j}$, $e_4 = \mathbf{k}$, and the coefficients $\mathrm{r}_\mu \in \mathbb{R}$.

The imaginary units of quaternions are represented by bold upright letters $\mathbf{i}, \mathbf{j}, \mathbf{k}$, whereas the imaginary unit in complex numbers is represented by an italic i, to avoid confusion with the quaternion units $\mathbf{i}, \mathbf{j}, \mathbf{k}$. It should be specifically noted that in physics, the symbols $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are commonly used as unit vectors in three-dimensional space to indicate specific directions. Conversely, in the abstract algebraic system of quaternions, these same symbols $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are assigned the role of imaginary units, forming the basis of the vector part of the quaternion, and adhering to distinct multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1 \tag{50}$$

The modulus of a Hamilton quaternion H is $|\mathrm{H}|$, and the square of its modulus is defined as the sum of the squares of its components, which is a non-negative real number:

$$|H|^2 = {r_1}^2 + {r_2}^2 + {r_3}^2 + {r_4}^2,\ |H| \in \mathbb{R} \geq 0 \tag{51}$$

In quaternion theory, these imaginary units help construct a conceptual framework analogous to physical spatial directions. However, this does not imply that the imaginary units $\mathbf{i}, \mathbf{j}, \mathbf{k}$ of a quaternion can be directly equated to the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ used to represent directions in physics. Although they possess a correspondence in geometric directionality, fundamental differences exist in their algebraic operation rules. This uniqueness in mathematical structure endows quaternions with a powerful ability to connect algebra with geometry. The biquaternions (quaternions with complex coefficients) to be discussed next encompass the imaginary unit i from the complex domain and the imaginary units $\mathbf{i}$ from quaternions. Although both i and $\mathbf{i}$ are defined as having a square equal to -1 within their respective mathematical systems, they are fundamentally different in their algebraic properties: the complex imaginary unit i satisfies the commutative law of multiplication and lies in the algebraic center; whereas the quaternion imaginary units $\mathbf{i}, \mathbf{j}, \mathbf{k}$ follow non-commutative multiplication rules. It is precisely this difference in algebraic structure that supports the rich physical connotations of biquaternion theory when describing nine-dimensional spacetime.

Hamilton quaternions successfully describe classical three-dimensional physical space; however, to describe a nine-dimensional spacetime incorporating electric potential degrees of freedom, it is necessary to extend the coefficient field from the real domain to the complex domain. The quaternion thus defined is termed a biquaternion, denoted as Q. A biquaternion Q can be viewed as a complex generalization of a Hamilton quaternion, with its coefficients z_μ being complex numbers.

$$Q = z_1 + z_2\mathbf{i} + z_3\mathbf{j} + z_4\mathbf{k} \tag{52}$$

$$Q = \sum_{\mu=1}^{4} z_\mu \, e_\mu \tag{53}$$

Where $e_1 = 1, e_2 = \mathbf{i}, e_3 = \mathbf{j}, e_4 = \mathbf{k}$, adhering to the quaternion multiplication rules (50). The coefficients $z_\mu \in \mathbb{C}$.

Because the square of the modulus of a biquaternion is $|Q|^2 = {z_1}^2 + {z_2}^2 + {z_3}^2 + {z_4}^2$, it is generally a complex number. However, the complex domain lacks an ordered nature (i.e., magnitudes cannot be directly compared), so to establish a physically meaningful metric, it is necessary to define the norm modulus of a biquaternion, denoted as $\|Q\|$:

$$\|Q\|^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 \tag{54}$$

This norm modulus definition ensures the positive-definiteness of the metric, guaranteeing physically that observational values of energy or distance are real numbers.

When the coefficients of the biquaternion Q change from complex to real numbers, it reduces to a real-coefficient quaternion H (Hamilton quaternion), namely:

$$\|Q\| = |H| \tag{55}$$

It is thus evident that mathematically, complex numbers, quaternions, and biquaternions [9] all have explicit definitions. Therefore, to physically extend Complex Electrodynamic Spacetime Relativity to Biquaternion Electrodynamic Spacetime Relativity, the physical quantities within the former must be extended to biquaternion physical quantities. For example, biquaternion velocity states, biquaternion position states, etc., and their corresponding reference frames will be collectively referred to as "biquaternion electrodynamic inertial reference frames."

Further, complex physical quantities can be generalized to physical quantities in the biquaternion domain, defining a quantity containing eight non-zero components as a complete biquaternion. Constructing biquaternion inertial frames Σ_{Q} and Σ'_{Q}, the position states of event P in these two reference frames are respectively R_{Q} and R'_{Q}, with norm moduli $\|R_{\mathrm{Q}}\|$ and $\|R'_{\mathrm{Q}}\|$. The eight-dimensional components of R_{Q} and R'_{Q} are respectively (F_1, F_2, X_1, X_2, Y_1, Y_2, Z_1, Z_2) and (F'_1, F'_2, X'_1, X'_2, Y'_1, Y'_2, Z'_1, Z'_2). t and t' represent the time in reference frames Σ_{Q} and Σ'_{Q}, respectively. Here, the eight-dimensional space plus one-dimensional time constitutes a nine-dimensional spacetime architecture. Suppose Σ'_{Q} moves relative to Σ_{Q} with a biquaternion velocity state V_{Q}. Its eight components are (V_1, V_2, V_{x1}, V_{x2}, V_{y1}, V_{y2}, V_{z1}, V_{z2}), and its norm modulus is $\|V_{\mathrm{Q}}\|$. The relevant physical quantities are defined as follows:

The complete biquaternion position state R_{Q} of the observing reference frame and its norm modulus $\|R_{\mathrm{Q}}\|$ are:

$$R_{\mathrm{Q}} = (F_1 + F_2 i) + (X_1 + X_2 i)\mathbf{i} + (Y_1 + Y_2 i)\mathbf{j} + (Z_1 + Z_2 i)\mathbf{k} \tag{56}$$

$$\|R_{\mathrm{Q}}\|^2 = F^2 + r^2 = \sum_{n=1}^{2} \left(F_n{}^2 + X_n{}^2 + Y_n{}^2 + Z_n{}^2\right) \quad (57)$$

Where, $F^2 = \sum_{n=1}^{2} F_n{}^2,\ r^2 = \sum_{n=1}^{2} \left(X_n{}^2 + Y_n{}^2 + Z_n{}^2\right)$

The complete biquaternion position state of the observed reference frame is R'_{Q} and its norm modulus is $\|R'_{\mathrm{Q}}\|$:

$$R'_{\mathrm{Q}} = (F'_1 + F'_2 i) + (X'_1 + X'_2 i)\mathbf{i} + (Y'_1 + Y'_2 i)\mathbf{j} + (Z'_1 + Z'_2 i)\mathbf{k} \quad (58)$$

$$\|R'_{\mathrm{Q}}\|^2 = F'^2 + r'^2 = \sum_{n=1}^{2} \left(F'^2_n + X'_n{}^2 + Y'^2_n + Z'_n{}^2\right) \quad (59)$$

Where, $F'^2 = \sum_{n=1}^{2} F'^2_n,\ r'^2 = \sum_{n=1}^{2} \left(X'_n{}^2 + Y'_n{}^2 + Z'_n{}^2\right)$

The complete biquaternion velocity state V_{Q} and its norm modulus $\|V_{\mathrm{Q}}\|$ are:

$$V_{\mathrm{Q}} = (V_1 + V_2 i) + (V_{x1} + V_{x2} i)\mathbf{i} + \left(V_{y1} + V_{y2} i\right)\mathbf{j} + (V_{z1} + V_{z2} i)\mathbf{k} \quad (60)$$

$$\|V_{\mathrm{Q}}\|^2 = V^2 + V_r{}^2 = \sum_{n=1}^{2}\left(V_n{}^2 + V_{xn}{}^2 + V_{yn}{}^2 + V_{zn}{}^2\right) \quad (61)$$

Where, $V^2 = \sum_{n=1}^{2} V_n{}^2,\ V_r{}^2 = \sum_{n=1}^{2} \left(V_{xn}{}^2 + V_{yn}{}^2 + V_{zn}{}^2\right)$

2.3.2 Fundamental Postulates of Complete Biquaternion Electrodynamic Spacetime Relativity

Based on the preceding context, physical quantities related to complex reference frames are extended to biquaternion physical quantities. Consequently, the two postulates (A5) and (A6) of Complex Electrodynamic Spacetime Relativity are extended to the postulates of Biquaternion Electrodynamic Spacetime Relativity.

• Postulate (A7) Principle of Relativity for Biquaternion Electrodynamic Spacetime: The laws of physics have the same mathematical form in all biquaternion electrodynamic inertial reference frames (i.e., physical laws remain covariant under biquaternion Lorentz transformations).

• Postulate (A8) Principle of the Invariance of the Speed of Light / Electric Potential Limit in Biquaternion Electrodynamic Spacetime: In any biquaternion electrodynamic inertial reference frame, the norm modulus of the biquaternion velocity state of light is an invariant constant C_0 (i.e., the inherent speed of light in classical vacuum), which does not change with the choice of reference frame.

Equivalently, the norm modulus of the biquaternion potential state is an invariant limit constant Φ_0, and it also does not change with the choice of reference frame.

It is thus evident that postulates (A7) and (A8) are not mere simple superpositions of the preceding postulates, but rather higher-dimensional logical generalizations that encompass and unify postulates (A1) through (A6). Under this axiomatic system, Biquaternion Electrodynamic Spacetime Relativity is a nine-dimensional spacetime relativity (i.e., an eight-dimensional biquaternion space and a one-dimensional time), which can naturally simplify to various lower-dimensional spacetime relativities. Special Relativity, Electric Potential Relativity, and Complex Electrodynamic Spacetime Relativity are all special cases of it.

It should be particularly noted that postulate (A8) can be viewed as a generalized principle of the constancy of the speed of light, indicating that in Electrodynamic Spacetime Relativity, the mass of a photon is zero, and the projection of the inherent speed of light C_0 onto a coordinate component—the coordinate speed of light C'_0—is less than C_0.

2.3.3 Derivation of Complete Biquaternion Electrodynamic Spacetime Relativity

Mathematically, the Electrodynamic Spacetime Relativity of complete biquaternions can be expanded into multiple special combinational forms, which can generally be classified into three types of special biquaternion electrodynamic spacetime relativities: Type A, Type B, and Type AB. They correspond to special biquaternion spaces and time, respectively, as follows:

The physical quantities corresponding to Type A biquaternion space are:

$$F'_1 = X'_2 = Y'_2 = Z'_2 = 0$$

$$R_Q = F_2 i + X_1\mathbf{i} + Y_1\mathbf{j} + Z_1\mathbf{k} \tag{62}$$
$$R'_Q = F'_2 i + X'_1\mathbf{i} + Y'_1\mathbf{j} + Z'_1\mathbf{k} \tag{63}$$
$$V_Q = V_2 i + V_{x1}\mathbf{i} + V_{y1}\mathbf{j} + V_{z1}\mathbf{k} \tag{64}$$

Time is merely a real scalar related to the Type A quaternion space and velocity. Analysis reveals that Type A special biquaternion spacetime is the familiar classical spacetime, corresponding to a five-dimensional spacetime consisting of a real electric potential (imaginary velocity state), real three-dimensional velocity, and time. The corresponding Type A special biquaternion electrodynamic relativity can be further simplified into the previously discussed complex relativity, Special Relativity, and Electric Potential Relativity.

The physical quantities corresponding to Type B biquaternion space are:

$$F'_2 = X'_1 = Y'_1 = Z'_1 = 0$$
$$R_Q = F_1 + X_2 i\mathbf{i} + Y_2 i\mathbf{j} + Z_2 i\mathbf{k} \tag{65}$$
$$R'_Q = F'_1 + X'_2 i\mathbf{i} + Y'_2 i\mathbf{j} + Z'_2 i\mathbf{k} \tag{66}$$
$$V_Q = V_1 + V_{x2} i\mathbf{i} + V_{y2} i\mathbf{j} + V_{z2} i\mathbf{k} \tag{67}$$

Time is merely a real scalar related to the Type B space and velocity. Type B biquaternion space forms a complex conjugate mirror relationship with the (Type A) spacetime discussed herein. Theoretically, this is termed "complex dual mirror spacetime," which is not only a requirement of mathematical structural symmetry but may also foreshadow the deep structure of physical reality.

Does this mirror spacetime exist merely to satisfy mathematical completeness and symmetry? Or does it portend an invisible companion, a physical world connected to the observable physical world via complex dimensions? This "Imaginary number reality" may share a profound geometric homology with the complex nature of the wavefunction in quantum mechanics; namely, quantum phenomena might be the projected outcome of high-dimensional biquaternion spacetime in lower-dimensional spacetime.

Although these questions are beyond the scope of this paper, the mathematical structure revealed by Biquaternion Electrodynamic Spacetime Relativity undoubtedly opens new pathways for future exploration of the unified interpretation of General Relativity and quantum mechanics, as well as dark matter and dark energy.

Naturally, in theory, numerous combinational possibilities exist between Type A and Type B biquaternions. These are collectively referred to herein as Type AB biquaternions. Type AB biquaternion spacetime is a richer and more profound physical spacetime that merits in-depth exploration.

Therefore, from the perspectives of both the symmetry of physical theory and the completeness of mathematical structure, it is necessary to extend the fundamental physical concepts and basic theoretical postulates of Electrodynamic Spacetime Relativity from complex numbers to complete biquaternions, constructing an Electrodynamic Spacetime Relativity in nine-dimensional spacetime. This not only encompasses existing spacetime theories (i.e., Special Relativity) but also possesses significant foresight and developmental potential.

To concisely and rigorously construct the algebraic foundation of high-dimensional Electrodynamic Spacetime Relativity, an inner product operation in biquaternion space is introduced herein, namely the biquaternion inner product $\langle R_Q, V_Q \rangle$.

Let $R_Q, V_Q \in Q$ be complete biquaternions, whose component coefficients in each dimension are complex numbers, denoted as $R_{Q\mu}, V_{Q\mu} \in \mathbb{C}$ (where $\mu = 1,2,3,4$). Therefore, they can be expanded as:

$$R_{Q1} = F_1 + F_2 i,\quad R_{Q2} = X_1 + X_2 i,\quad R_{Q3} = Y_1 + Y_2 i,\quad R_{Q4} = Z_1 + Z_2 i$$
$$V_{Q1} = V_1 + V_2 i,\quad V_{Q2} = V_{x1} + V_{x2} i,\quad V_{Q3} = V_{y1} + V_{y2} i,\quad V_{Q4} = V_{z1} + V_{z2} i$$

The underlying biquaternion Hermitian inner product is defined as the sum of the conjugate products of the corresponding complex components:

$$\langle R_{\mathrm{Q}}, V_{\mathrm{Q}} \rangle := \sum_{\mu=1}^{4} \left(R_{\mathrm{Q}\mu}^{\dagger} V_{\mathrm{Q}\mu} \right) \in \mathbb{C} \tag{68}$$

To maintain consistency with the "dot product" notation in the low-dimensional complex plane and classical physics, it is stipulated that the "electrodynamic real scalar product" for high-dimensional biquaternions continues to be denoted by the symbol "·". For the biquaternion velocity state V_{Q}, its physical attributes map to the real axis; thus, the real scalar product of R_{Q} and V_{Q} is defined as the real part of this biquaternion Hermitian inner product:

$$R_{\mathrm{Q}} \cdot V_{\mathrm{Q}} := \mathrm{Re}\langle R_{\mathrm{Q}}, V_{\mathrm{Q}} \rangle = \sum_{\mu=1}^{4} \mathrm{Re}\left(R_{\mathrm{Q}\mu}^{\dagger} V_{\mathrm{Q}\mu} \right) \in \mathbb{R} \tag{69}$$

Upon expansion, this yields the sum of the products of the respective real coefficients:

$$R_{\mathrm{Q}} \cdot V_{\mathrm{Q}} = F_1 V_1 + F_2 V_2 + X_1 V_{x1} + X_2 V_{x2} + Y_1 V_{y1} + Y_2 V_{y2} + Z_1 V_{z1} + Z_2 V_{z2} \tag{70}$$

Analyzing the establishment process of Complex Electrodynamic Spacetime Relativity reveals an intrinsic logical isomorphism. Just as the three-dimensional Special Relativity vector equation in Reference [8] was dimensionally reduced to a two-dimensional form and transformed into a complex equation, this vector equation can similarly be extended from a three-dimensional to an eight-dimensional orthogonal real vector space. By substituting the real unit vectors with biquaternion basis elements, the Biquaternion Electrodynamic Spacetime Relativity (ESR) equations containing eight components are ultimately derived.

$$\boldsymbol{R}'_{\mathrm{Q}} = \boldsymbol{F}'_1 + \boldsymbol{F}'_2 + \boldsymbol{X}'_1 + \boldsymbol{X}'_2 + \boldsymbol{Y}'_1 + \boldsymbol{Y}'_2 + \boldsymbol{Z}'_1 + \boldsymbol{Z}'_2 \tag{71}$$

$$\boldsymbol{R}_{\mathrm{Q}} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{X}_1 + \boldsymbol{X}_2 + \boldsymbol{Y}_1 + \boldsymbol{Y}_2 + \boldsymbol{Z}_1 + \boldsymbol{Z}_2 \tag{72}$$

$$\boldsymbol{V}_{\mathrm{Q}} = \boldsymbol{V}_1 + \boldsymbol{V}_2 + \boldsymbol{V}_{x1} + \boldsymbol{V}_{x2} + \boldsymbol{V}_{y1} + \boldsymbol{V}_{y2} + \boldsymbol{V}_{z1} + \boldsymbol{V}_{z2} \tag{73}$$

The modulus $|\boldsymbol{V}_{\mathrm{Q}}|$ of the eight-dimensional real vector velocity $\boldsymbol{V}_{\mathrm{Q}}$ is:

$$|\boldsymbol{V}_{\mathrm{Q}}|^2 = {V_1}^2 + {V_2}^2 + {V_{x1}}^2 + {V_{x2}}^2 + {V_{y1}}^2 + {V_{y2}}^2 + {V_{z1}}^2 + {V_{z2}}^2 \tag{74}$$

$$\boldsymbol{R}_{\mathrm{Q}} \cdot \boldsymbol{V}_{\mathrm{Q}} = F_1 V_1 + F_2 V_2 + X_1 V_{x1} + X_2 V_{x2} + Y_1 V_{y1} + Y_2 V_{y2} + Z_1 V_{z1} + Z_2 V_{z2} \tag{75}$$

The two-dimensional real vector relativistic Eqs. (27) and (28) are extended into relativistic equations for an eight-dimensional real vector space:

$$\boldsymbol{R}'_{\mathrm{Q}} = \boldsymbol{R}_{\mathrm{Q}} + \left(\gamma(\boldsymbol{V}_{\mathrm{Q}}) - 1 \right) \frac{(\boldsymbol{R}_{\mathrm{Q}} \cdot \boldsymbol{V}_{\mathrm{Q}}) \boldsymbol{V}_{\mathrm{Q}}}{|\boldsymbol{V}_{\mathrm{Q}}|^2} - \gamma(\boldsymbol{V}_{\mathrm{Q}}) \boldsymbol{V}_{\mathrm{Q}} t \tag{76}$$

$$t' = \gamma(\boldsymbol{V}_{\mathrm{Q}}) \left(t - \frac{\boldsymbol{R}_{\mathrm{Q}} \cdot \boldsymbol{V}_{\mathrm{Q}}}{{C_0}^2} \right) \tag{77}$$

Where, $\gamma(\boldsymbol{V}_{\mathrm{Q}}) = \frac{1}{\sqrt{1 - (|\boldsymbol{V}_{\mathrm{Q}}|/C_0)^2}}$

The aforementioned vector equations are expanded into eight mutually orthogonal axial vector equations; the axial unit vectors for each equation are then removed, transforming the eight vector equations into eight scalar equations. These eight scalar equations are then respectively multiplied by the corresponding biquaternion basis elements $\mathrm{k}_n \in \{1, i, \mathbf{i}, i\mathbf{i}, \mathbf{j}, i\mathbf{j}, \mathbf{k}, i\mathbf{k}\}$, followed by equivalent substitutions: Replace the real vector dot product $\boldsymbol{R}_{\mathrm{Q}} \cdot \boldsymbol{V}_{\mathrm{Q}}$ with the dot product $R_{\mathrm{Q}} \cdot V_{\mathrm{Q}}$ defined by the real part of the Hermitian inner product of the two quaternions.

and the biquaternion norm modulus $\|V_{\mathrm{Q}}\|$ replaces the real vector modulus $|\boldsymbol{V}_{\mathrm{Q}}|$. Thus, equations for the eight biquaternion components are obtained as follows:

$$\mathrm{k_n} R'_{\mathrm{Q}n} = \mathrm{k_n} \left[R_{\mathrm{Q}n} + \left(\gamma(V_{\mathrm{Q}}) - 1 \right) \frac{(R_{\mathrm{Q}} \cdot V_{\mathrm{Q}}) V_{\mathrm{Q}n}}{\|V_{\mathrm{Q}}\|^2} - \gamma(V_{\mathrm{Q}}) V_{\mathrm{Q}n} t \right] \tag{78}$$

Where, $\gamma(V_{\mathrm{Q}}) = \frac{1}{\sqrt{1 - (\|V_{\mathrm{Q}}\|/C_0)^2}}$

In this equation, the 8 basis elements of the biquaternion are: $\mathrm{k_n} \in \{1, i, \mathbf{i}, i\mathbf{i}, \mathbf{j}, i\mathbf{j}, \mathbf{k}, i\mathbf{k}\}$. The 8 components of the biquaternion position state R_Q corresponding to the basis elements $\mathrm{k_n}$ are: $R_{\mathrm{Q}n} \in \{F_1, F_2, X_1, X_2, Y_1, Y_2, Z_1, Z_2\}$. The 8 components of the biquaternion position state R'_Q corresponding to the basis elements $\mathrm{k_n}$ are: $R'_{\mathrm{Q}n} \in \{F'_1, F'_2, X'_1, X'_2, Y'_1, Y'_2, Z'_1, Z'_2\}$. The 8 components of the biquaternion velocity state V_Q corresponding to the basis elements $\mathrm{k_n}$ are: $V_{\mathrm{Q}n} \in \{V_1, V_2, V_{x1}, V_{x2}, V_{y1}, V_{y2}, V_{z1}, V_{z2}\}$.

By summing the aforementioned eight biquaternion component equations, the fundamental equation of Electrodynamic Spacetime Relativity for a biquaternion velocity state, namely the generalized Lorentz transformation, is derived:

$$R'_\mathrm{Q} = R_\mathrm{Q} + \left(\gamma(V_\mathrm{Q}) - 1\right)\frac{(R_\mathrm{Q}\cdot V_\mathrm{Q})V_\mathrm{Q}}{\|V_\mathrm{Q}\|^2} - \gamma(V_\mathrm{Q})V_\mathrm{Q}t \tag{79}$$

$$t' = \gamma(V_\mathrm{Q})\left(t - \frac{R_\mathrm{Q}\cdot V_\mathrm{Q}}{C_0{}^2}\right) \tag{80}$$

Where, $\gamma(V_\mathrm{Q}) = \frac{1}{\sqrt{1-(\|V_\mathrm{Q}\|/C_0)^2}}$

By comparing the transformation equations in real vector form (76) (77) with those in biquaternion form (79) (80), a strict mathematical equivalence between the two can be established. This conclusion not only strongly verifies the self-consistency of the derivation process but also bears profound physical significance: it indicates that biquaternion algebra is not merely a mathematical simplification tool, but the most natural and intrinsic language for describing high-dimensional non-linear spacetime geometry. Compared to traditional real vector component expressions, the biquaternion structure utilizes its unique algebraic closure to unify complex spacetime rotations and Lorentz-like transformations into a compact and elegant geometric algebraic framework.

In the complete biquaternion electrodynamic spacetime reference frame, the fundamental postulates (A7) and (A8) of Electrodynamic Spacetime Relativity hold true. Therefore, the concept of "invariant spacetime interval" from Special Relativity can be naturally extended to the complete biquaternion form. Its core requirement is that the inherent speed of light C_0 under this theory remains invariant, which must satisfy:

$$\|R'_\mathrm{Q}\|^2 - C_0^2 t'^2 = \|R_\mathrm{Q}\|^2 - C_0^2 t^2 \tag{81}$$

Expanding the above equation yields:

$$\begin{aligned} &F_1'^2 + F_2'^2 + X_1'^2 + X_2'^2 + Y_1'^2 + Y_2'^2 + Z_1'^2 + Z_2'^2 - C_0{}^2 t'^2 \\ =\ &F_1{}^2 + F_2{}^2 + X_1{}^2 + X_2{}^2 + Y_1{}^2 + Y_2{}^2 + Z_1{}^2 + Z_2{}^2 - C_0{}^2 t^2 \end{aligned} \tag{82}$$

Dividing both sides of Eq. (78) simultaneously by $\mathrm{k_n}$ yields 8 scalar equations; substituting these 8 scalar equations together with the time transformation Eq. (80) into the left side of the spacetime interval Eq. (82), and following rigorous algebraic operations, it can be proven that this expression is identically equal to the right side of the equation. This confirms the self-consistency of Complete Biquaternion Electrodynamic Spacetime Relativity in its mathematical structure—the spacetime interval remains invariant under Biquaternion Electrodynamic Spacetime Relativity. Simultaneously, Eq. (82) clearly indicates that the spatial part (including the complex space) is internally Euclidean (positive-definite), whereas the relationship between the overall space and time is Minkowskian (hyperbolic).

2.3.4 Special Forms and Structural Analysis of the Fundamental Equations of Electrodynamic Spacetime Relativity

1) Collinear and Orthogonal Decomposition Forms of the Fundamental Equations

To intuitively understand the physical and geometric significance of the biquaternion Lorentz transformation, the biquaternion position state R_Q is decomposed into a position state component $R_\parallel$ parallel to the velocity state V_Q, and a position state component $R_\perp$ perpendicular to the velocity state.

According to the definitions in vector algebra and the mapping relationship between eight-dimensional vectors and biquaternions (eight-dimensional) established in the previous section, the parallel component $R_{\parallel}$ and the perpendicular component $R_{\perp}$ are respectively:

$$R_{\parallel} = \frac{(R_{\mathrm{Q}} \cdot V_{\mathrm{Q}})V_{\mathrm{Q}}}{\|V_{\mathrm{Q}}\|^2},\ R_{\perp} = R_{\mathrm{Q}} - R_{\parallel} \tag{83}$$

Obviously satisfying $R_{\perp} \cdot V_{\mathrm{Q}} = 0,\quad R_{\perp} = R_{\mathrm{Q}} - R_{\parallel}$.

Substituting the aforementioned decomposition into the transformation Eq. (79):

$$R'_{\mathrm{Q}} = (R_{\parallel} + R_{\perp}) + \left(\gamma(V_{\mathrm{Q}}) - 1\right)R_{\parallel} - \gamma(V_{\mathrm{Q}})V_{\mathrm{Q}}t \tag{84}$$

Rearranging and combining like terms yields:

$$R'_{\mathrm{Q}} = R_{\perp} + \gamma(V_{\mathrm{Q}})\left(R_{\parallel} - V_{\mathrm{Q}}t\right) \tag{85}$$

Consequently, separated transformation equations are obtained:

• Transverse Component:

$$R'_{\perp} = R_{\perp} \tag{86}$$

• Longitudinal Component:

$$R'_{\parallel} = \gamma(V_{\mathrm{Q}})\left(R_{\parallel} - V_{\mathrm{Q}}t\right) \tag{87}$$

This is formally completely identical to the transformation equations for one-dimensional motion in Special Relativity (noting that V_{Q} here is a generalized biquaternion velocity, containing the spatial velocity and the imaginary speed corresponding to the electric potential).

• Time Transformation: Utilizing $R_{\mathrm{Q}} \cdot V_{\mathrm{Q}} = R_{\parallel} \cdot V_{\mathrm{Q}}$, Eq. (80) can be rewritten as:

$$t' = \gamma(V_{\mathrm{Q}})\left(t - \frac{R_{\parallel} \cdot V_{\mathrm{Q}}}{{C_0}^2}\right) \tag{88}$$

Where, $\gamma(V_{\mathrm{Q}}) = \frac{1}{\sqrt{1 - (\|V_{\mathrm{Q}}\|/C_0)^2}}$

The above decomposition demonstrates that Biquaternion Electrodynamic Spacetime Relativity strictly adheres to the properties of the Lorentz group in its mathematical structure. All relativistic effects (e.g., length contraction, time dilation, and the relativity of simultaneity) occur exclusively in the dimension collinear with the generalized velocity state V_{Q}, while orthogonal dimensions remain invariant.

2) Hyperbolic Function Expression of the Fundamental Equations

To profoundly reveal the spacetime geometric structure of Electrodynamic Spacetime Relativity, the Generalized Rapidity φ is introduced, rewriting the collinear transformation Eqs. (87) (88) into a perfectly symmetric hyperbolic rotation form.

Introducing the generalized rapidity φ, its relationship with the norm modulus of the generalized velocity is:

$$\frac{\|V_{\mathrm{Q}}\|}{C_0} = \tanh\varphi\ ,\ \ \gamma(V_{\mathrm{Q}}) = \frac{1}{\sqrt{1 - (\|V_{\mathrm{Q}}\|/C_0)^2}} = \cosh\varphi,\ \ \gamma(V_{\mathrm{Q}})\frac{\|V_{\mathrm{Q}}\|}{C_0} = \sinh\varphi \tag{89}$$

Utilizing the aforementioned relationships, the transformation Eqs. (87) (88) can be rewritten as the following perfectly symmetric system of biquaternion hyperbolic rotation equations:

$$R'_{\parallel} = R_{\parallel}\cosh\varphi - \left(\frac{V_{\mathrm{Q}}}{\|V_{\mathrm{Q}}\|}\ C_0 t\right)\ \sinh\varphi \tag{90}$$

$$C_0 t' = C_0 t\cosh\varphi - \left(\frac{R_{\parallel} \cdot V_{\mathrm{Q}}}{\|V_{\mathrm{Q}}\|}\right)\sinh\varphi \tag{91}$$

2.3.5 Electric Potential State Expression of the Fundamental Equations of Electrodynamic Spacetime Relativity

Based on the underlying geometric duality between velocity and electric potential, the biquaternion velocity state can be strictly transformed into the biquaternion potential state. According to Eq. (9), the real electric potential corresponds to the imaginary velocity; thus, the biquaternion potential state ϕ_Q is defined as follows:

$$\phi_\mathrm{Q} := (\phi_1 i + \phi_2) + (\phi_{x1} i + \phi_{x2})\mathbf{i} + (\phi_{y1} i + \phi_{y2})\mathbf{j} + (\phi_{z1} i + \phi_{z2})\mathbf{k} \tag{92}$$

Extending the transformation Eq. (21) of complex velocity and complex potential to high-dimensional space yields the transformation relationship between the biquaternion velocity state and the biquaternion potential state:

$$V_\mathrm{Q} = \psi\phi_\mathrm{Q} = i\frac{C_0}{\Phi_0}\left[(\phi_1 i + \phi_2) + (\phi_{x1} i + \phi_{x2})\mathbf{i} + (\phi_{y1} i + \phi_{y2})\mathbf{j} + (\phi_{z1} i + \phi_{z2})\mathbf{k}\right] \tag{93}$$

$$\|V_\mathrm{Q}\| = \frac{C_0}{\Phi_0}\|\phi_\mathrm{Q}\| \tag{94}$$

Comparing the coefficients of the terms in Eq. (93) and Eq. (60) gives:

$$V_1 = -\frac{C_0}{\Phi_0}\phi_1,\ V_{x1} = -\frac{C_0}{\Phi_0}\phi_{x1},\ V_{y1} = -\frac{C_0}{\Phi_0}\phi_{y1}, V_{z1} = -\frac{C_0}{\Phi_0}\phi_{z1}$$

$$V_2 = \frac{C_0}{\Phi_0}\phi_2,\ V_{x2} = \frac{C_0}{\Phi_0}\phi_{x2},\ V_{y2} = \frac{C_0}{\Phi_0}\phi_{y2}, V_{z2} = \frac{C_0}{\Phi_0}\phi_{z2}$$

Substituting the above components into the expansion of the real scalar product of the biquaternion velocity state (70) yields:

$$R_Q \cdot V_Q = \frac{C_0}{\Phi_0}\left(-F_1\phi_1 + F_2\phi_2 - X_1\phi_{x1} + X_2\phi_{x2} - Y_1\phi_{y1} + Y_2\phi_{y2} - Z_1\phi_{z1} + Z_2\phi_{z2}\right) \tag{95}$$

According to the unified definition principle established previously, for the biquaternion potential state ϕ_Q, its physical attributes map to the imaginary axis. Therefore, the "electrodynamic real scalar product" of the biquaternion position state R_Q and the potential state ϕ_Q is strictly defined as the negative imaginary part of their underlying biquaternion Hermitian inner product:

$$R_\mathrm{Q} \cdot \phi_\mathrm{Q} := -\mathrm{Im}\langle R_\mathrm{Q}, \phi_\mathrm{Q}\rangle = -\sum_{\mu=1}^{4} \mathrm{Im}\left(R_{\mathrm{Q}\mu}^\dagger \phi_{\mathrm{Q}\mu}\right) \in \mathbb{R} \tag{96}$$

Upon expansion, the corresponding product relationship of the real coefficients is:

$$R_\mathrm{Q} \cdot \phi_\mathrm{Q} = \left(-F_1\phi_1 + F_2\phi_2 - X_1\phi_{x1} + X_2\phi_{x2} - Y_1\phi_{y1} + Y_2\phi_{y2} - Z_1\phi_{z1} + Z_2\phi_{z2}\right) \tag{97}$$

Combining Eqs. (95) and (97), it can be seen that the real scalar products of the biquaternion velocity state and the potential state also strictly maintain the following algebraic equivalence relationship:

$$R_\mathrm{Q} \cdot V_\mathrm{Q} = \frac{C_0}{\Phi_0} R_\mathrm{Q} \cdot \phi_\mathrm{Q} \tag{98}$$

Substituting the transformation Eq. (93), the norm relationship (94), and the real scalar product equivalence relationship (98) respectively into the fundamental equations of Electrodynamic Spacetime Relativity expressed in the biquaternion velocity state, the fundamental equations expressed in the biquaternion potential state can be naturally derived, similarly, other forms of the fundamental equations for the electric potential state can be derived as follows:

1) Fundamental equations of Electrodynamic Spacetime Relativity expressed in the potential state:

$$R'_\mathrm{Q} = R_\mathrm{Q} + i\left(\gamma(\phi_\mathrm{Q}) - 1\right)\frac{(R_\mathrm{Q}\cdot\phi_\mathrm{Q})\phi_\mathrm{Q}}{\|\phi_\mathrm{Q}\|^2} - i\gamma(\phi_\mathrm{Q})\frac{C_0}{\Phi_0}\phi_\mathrm{Q} t \tag{99}$$

$$t' = \gamma(\phi_\mathrm{Q})\left(t - \frac{R_\mathrm{Q}\cdot\phi_\mathrm{Q}}{\Phi_0 C_0}\right) \tag{100}$$

$$\gamma(\phi_\mathrm{Q}) = \frac{1}{\sqrt{1-\left(\|\phi_\mathrm{Q}\|/\Phi_0\right)^2}} \tag{101}$$

Therefore, theoretically, Eqs. (79) and (80) and Eqs. (99) and (100) are completely equivalent.

2) Collinear and orthogonal decomposition forms of the fundamental equations expressed in the potential state:

$$R'_\perp = R_\perp$$

$$R'_{\parallel} = \gamma(\phi_{\mathrm{Q}})\left(R_{\parallel} - i\frac{C_0}{\Phi_0}\phi_{\mathrm{Q}}t\right) \tag{102}$$

$$t' = \gamma(\phi_{\mathrm{Q}})\left(t - \frac{R_{\parallel}\cdot\phi_{\mathrm{Q}}}{\Phi_0 C_0}\right) \tag{103}$$

Where, $\gamma(\phi_{\mathrm{Q}}) = \frac{1}{\sqrt{1-(\|\phi_{\mathrm{Q}}\|/\Phi_0)^2}}$

3) The hyperbolic function form of the fundamental equations is expressed in the potential state:

Because $V_{\mathrm{Q}} = i\frac{C_0}{\Phi_0}\phi_{\mathrm{Q}}$, from Eq. (89), the generalized rapidity φ defined by the electric potential can be obtained, satisfying:

$$\tanh\varphi = \frac{\|\phi_{\mathrm{Q}}\|}{\Phi_0}, \quad \gamma(\phi_{\mathrm{Q}}) = \frac{1}{\sqrt{1-(\|\phi_{\mathrm{Q}}\|/\Phi_0)^2}} = \cosh\varphi, \quad \gamma(\phi_{\mathrm{Q}})\frac{\|\phi_{\mathrm{Q}}\|}{\Phi_0} = \sinh\varphi$$

$$R'_{\parallel} = R_{\parallel}\cosh\varphi - i\left(\frac{\phi_{\mathrm{Q}}}{\|\phi_{\mathrm{Q}}\|}C_0 t\right)\sinh\varphi \tag{104}$$

$$C_0 t' = C_0 t\cosh\varphi - \left(\frac{R_{\parallel}\cdot\phi_{\mathrm{Q}}}{\|\phi_{\mathrm{Q}}\|}\right)\sinh\varphi \tag{105}$$

Therefore, in theoretical essence, the dynamical Eqs. (79), (80) expressed in the velocity state are completely equivalent to the field Eqs. (99), (100) expressed in the potential state. This mathematical duality profoundly reveals the core concept of Electrodynamic Spacetime Relativity: the motion of matter (velocity) and the distribution of the field (potential) are, in fact, projections of the same physical reality across different observational dimensions.

2.3.6 Fundamental Physical Effects of Biquaternion Electrodynamic Spacetime Relativity

Based on the fundamental Eqs. (86), (87), and (88) in the collinear decomposition form derived in Section 2.3.4, it is found that: in the dimension collinear with the generalized velocity state V_{Q}, the spacetime transformation strictly follows the form of the Lorentz transformation. Utilizing this isomorphism, the three major fundamental kinematic effects of Electrodynamic Spacetime Relativity can be directly derived.

2.3.6.1 Superposition Effect (Collinear Velocity/Potential)

In Electrodynamic Spacetime Relativity, when the relative motion directions of two reference frames are collinear, the superposition of their generalized velocities also follows the same hyperbolic geometric laws as in Special Relativity, as shown in Eqs. (90) (91).

Suppose the generalized velocity of the reference frame Σ_2 relative to Σ_1 is V_{Q1}, and the generalized velocity of the reference frame Σ_3 relative to Σ_2 is V_{Q2}, with V_{Q1} and V_{Q2} being collinear.

Because rapidity possesses linear additivity in collinear Lorentz transformations, i.e., $\varphi_{\mathrm{total}} = \varphi_1 \pm \varphi_2$, the synthesized generalized velocity V_{total} and rapidity satisfy Eq.(89): $\frac{\|V_{\mathrm{total}}\|}{C_0} = \tanh\varphi_{\mathrm{total}}$, and $\tanh(\varphi_1) = \frac{\|V_{\mathrm{Q1}}\|}{C_0}$, $\tanh(\varphi_2) = \frac{\|V_{\mathrm{Q2}}\|}{C_0}$.

$$\tanh\varphi_{\mathrm{total}} = \tanh(\varphi_1 \pm \varphi_2) = \frac{\tanh(\varphi_1) \pm \tanh(\varphi_2)}{1 \pm \tanh(\varphi_1)\tanh(\varphi_2)} \tag{106}$$

The " $\pm$ " sign depends on whether the two generalized velocities (or potential differences) are in the same or opposite directions.

Converting this to the norm form of the generalized velocity state yields the superposition formula for biquaternion velocity:

$$\|V_{\mathrm{total}}\| = \frac{\|V_{\mathrm{Q1}}\| \pm \|V_{\mathrm{Q2}}\|}{1 \pm \frac{\|V_{\mathrm{Q1}}\|\|V_{\mathrm{Q2}}\|}{{C_0}^2}} \tag{107}$$

This naturally leads to the collinear velocity superposition equation of Special

Relativity.

Based on the correspondence between biquaternion potential and biquaternion velocity, the superposition formula for biquaternion potential is obtained from (107):

$$\|\phi_{\text{total}}\| = \frac{\|\phi_{Q1}\| \pm \|\phi_{Q2}\|}{1 \pm \frac{\|\phi_{Q1}\|\|\phi_{Q2}\|}{\Phi_0{}^2}} \tag{108}$$

Eqs. (107) and (108) indicate that, regardless of how spatial velocities are superposed, or how spatial velocities and potential differences are combined, the norm modulus of the synthesized generalized velocity can never exceed the speed of light C_0 / electric potential limit Φ_0. This once again verifies the self-consistency of Postulate (A8).

2.3.6.2 Time Dilation Effect

Consider a clock in a stationary frame Σ. The clock's position remains unchanged, and the time interval it records is the proper time $\Delta\tau$. Therefore, the proper time is equal to the coordinate time Δt of the stationary reference frame, i.e., $\Delta t = \Delta\tau$. For this clock, its position coordinates in the Σ reference frame do not change with time, meaning the increment of the collinear electrodynamic position state is $\Delta R_{\|} = 0$, i.e.,$\Delta(R_{\|} \cdot V_Q) = 0$.

According to the transformation formula of Eq. (88): $\Delta t' = \gamma(V_Q)[\Delta t - \Delta(R_{\|} \cdot V_Q)/C_0^2]$, and substituting $\Delta(R_{\|} \cdot V_Q) = 0$, the time interval $\Delta t'$ observed by the moving frame Σ' can be obtained:

$$\Delta t' = \gamma(V_Q)\Delta t = \frac{\Delta t}{\sqrt{1-(\|V_Q\|/C_0)^2}} \tag{109}$$

Since$\gamma(V_Q) > 1$, therefore $\Delta t' > \Delta t$.

This indicates that as long as a generalized velocity V_Q exists for the reference frame (possessing both high-speed relative motion and a high potential difference simultaneously), the time of the clock in the generalized motion state runs slower than the time of the clock in the stationary state (time dilation).

When V_Q is solely contributed by the spatial velocity V, $\|V_Q\|^2 = |V|^2 = V_{x1}{}^2 + V_{y1}{}^2 + V_{z1}{}^2$, and the time dilation equation of Special Relativity is obtained:

$$\Delta t' = \gamma(V)\Delta t = \frac{\Delta t}{\sqrt{1-(|V|/C_0)^2}} \tag{110}$$

When V_Q is solely contributed by the electric potential ϕ, $\|V_Q\|^2 = \left(\frac{C_0}{\Phi_0}|\phi|\right)^2$, and the time dilation equation of Electric Potential Relativity is obtained:

$$\Delta t' = \gamma(\phi)\Delta t = \frac{\Delta t}{\sqrt{1-(|\phi|/\Phi_0)^2}} \tag{111}$$

2.3.6.3 Spatial Contraction Effect

Consider a "generalized ruler" stationary in the moving frame Σ' (which includes the real physical length related to real velocity, as well as the imaginary length related to the potential, i.e., the imaginary velocity state). Its proper length in the collinear direction is $l = \Delta\|R'_{\|}\|$. An observer in the stationary frame Σ measures both ends of this ruler simultaneously at $\Delta t = 0$.

Utilizing the transformation Eq. (87), $\Delta R'_{\|} = \gamma(V_Q)(\Delta R_{\|} - V_Q\Delta t)$, and substituting $\Delta t = 0$, the relationship between the measured length $l' = \Delta R_{\|}$ and the proper length can be obtained as $l = \gamma(V_Q)l'$, namely:

$$l' = \frac{l}{\gamma(V_Q)} = l\sqrt{1-(\|V_Q\|/C_0)^2} \tag{112}$$

This reveals the "generalized length contraction" effect. The following are its special cases:

Length contraction in Special Relativity: The described space $l', l,$ and velocity V in

Special Relativity are all real numbers, and the physical length along the direction of motion undergoes contraction. Namely, the length contraction effect in Special Relativity is:

$$l' = \frac{l}{\gamma(V_Q)} = l\sqrt{1-(\|V\|/C_0)^2} \tag{113}$$

Potential spatial contraction: In Electric Potential Relativity, the potential ϕ serves as the "imaginary velocity state" of the imaginary dimension, and the corresponding imaginary coordinates iF and iF' respectively will induce a relativistic contraction effect of the imaginary spatial length:

$$il' = \frac{il}{\gamma(\phi)} = il\sqrt{1-(\|\phi\|/\Phi_0)^2} \tag{114}$$

Imaginary length is an immeasurable physical quantity; whether it corresponds to phase space requires further in-depth research. It must be emphasized that in Electric Potential Relativity, the length of real physical space remains invariant, because the real space is orthogonal to the imaginary space.

By introducing the electric potential limit constant, this chapter has extended Special Relativity to complex and biquaternion forms, establishing a self-consistent fundamental theory of nine-dimensional spacetime—Electrodynamic Spacetime Relativity. This lays the foundation for the next chapter, "Nonlinear Maxwell's equations." The latter will be elaborated upon as a practical application of the special case of Electric Potential Relativity within this theory.

3. Nonlinear Maxwell's Equations

To resolve the problem of point charge energy divergence in electrodynamics, the electric potential limit constant is proposed herein. The introduction of this constant leads to the development of spacetime theory, namely, Electrodynamic Spacetime Relativity. Electrodynamic Spacetime Relativity reveals a special relativity symmetric to Special Relativity, namely Electric Potential Relativity. This chapter will discuss in depth the covariant relationship between Electric Potential Relativity and Maxwell's equations. A nonlinear Maxwell's equation is constructed to resolve the intractable problem of point charge energy divergence and to reveal new physical effects.

3.1 Derivation of Nonlinear Maxwell's Equations

The Electric Potential Relativity Eqs. (43) (45) share exactly the same mathematical form as the Lorentz transformation equations in Special Relativity; therefore, they also possess hyperbolic function expressions of the same form. It is worth noting that corresponding to the electric potential ϕ (imaginary velocity state) are the imaginary coordinate lengths F' and F, namely $R'_{\parallel} = i\ F'$ and $R_{\parallel} = i\ F$, while corresponding to the real velocity are the real coordinate lengths $R'_{\perp} = r'$ and $R_{\perp} = r$. The function relating the electric potential ϕ to the hyperbolic function angle φ simplifies to:

$$\gamma(\phi) = \cosh(\varphi) = \frac{1}{\sqrt{1-\left(\frac{\phi}{\Phi_0}\right)^2}} \tag{115}$$

$$\cosh^2(\varphi) - \sinh^2(\varphi) = 1 \tag{116}$$

From Eqs. (115) and (116), it follows that:

$$\tanh(\varphi) = \frac{\phi}{\Phi_0} \tag{117}$$

From Eqs. (102) and (103), it follows that:

$$i\ F' = i\ F\cosh(\varphi) - i\left(\frac{\phi}{\|\phi\|}C_0 t\right)\sinh(\varphi) \tag{118}$$

$$C_0 t' = -\frac{F\phi}{\|\phi\|}\sinh(\varphi) + C_0 t\cosh(\varphi) \tag{119}$$

$$r' = r \tag{120}$$

Eq. (120) indicates that in any equipotential reference frame, the lengths of the three-dimensional real space r′ and r are invariant. This is because, in Complex Electrodynamic Spacetime Relativity, the real space axes are orthogonal to the imaginary space axes.

Let $\varphi = \varphi_1 + \varphi_2$, $\frac{\phi_1}{\Phi_0} = \tanh(\varphi_1)$, and $\frac{\phi_2}{\Phi_0} = \tanh(\varphi_2)$. Herein, three equipotential reference frames, A, B, and C, exist. Assuming the electric potential of reference frame A is zero, the potential of reference frame B relative to reference frame A is ϕ_1, and the potential of reference frame C relative to reference frame B is ϕ_2. ϕ is the potential of reference frame C relative to reference frame A, which is the total potential after the superposition of potentials ϕ_1 and ϕ_2.

$$\frac{\phi}{\Phi_0} = \tanh(\varphi_1 + \varphi_2) \tag{121}$$

The superposition formula for the electric potential is obtained:

$$\phi = \frac{\phi_2 + \phi_1}{1 + \frac{\phi_1 \phi_2}{{\Phi_0}^2}} \tag{122}$$

The above equation indicates that the superposition of electric potentials is nonlinear, and a maximum value Φ_0 exists for the electric potential, meaning the electric potential has a limit. Eq. (122) presents a conclusion different from classical electrodynamics. However, when the electric potential is much smaller than the limit electric potential constant Φ_0, Eq. (122) simplifies to the linear superposition equation of electric potential in classical electrodynamics, $\phi = \phi_2 + \phi_1$. Evidently, according to Electric Potential Relativity, classical electrodynamics must be correspondingly extended.

The multiple equations above involve various hyperbolic functions; thus, the hyperbolic angle plays a crucial role. What exactly is the physical significance of the hyperbolic function angle φ? This will be further discussed through a thought experiment:

In a series circuit of n identical batteries, let the electric potential of the negative terminal of the first battery be zero. Suppose an ideal voltmeter exists, capable of accurately measuring true voltages. In the series circuit, two methods exist to measure the potential difference of the i-th battery:

The first method is to separately measure the potentials ϕ_i and ϕ_{i+1} of the negative and positive terminals of the battery relative to the zero-potential point, assuming that the difference between these two potentials is the voltage of the battery, $\Delta\phi_i = \phi_{i+1} - \phi_i$. Because the superposition of voltages is nonlinear, this voltage is related to the reference frame potential ϕ of the battery in the circuit, and the potential difference $\Delta\phi_i$ of the battery is a variable. Furthermore, $\Delta\phi_i > \Delta\phi_{i+1}$.

The second method is to directly measure the potential difference between the positive and negative terminals of any single battery using the voltmeter, which yields U_0, where U_0 is a constant independent of the reference frame potential ϕ of the battery in the circuit. According to existing electromagnetic theory, the voltage of a battery series circuit superposes linearly, thus yielding:

$$U_n = nU_0 \tag{123}$$

Where U_n is the total voltage of the battery series circuit, and U_n is referred to as the linear voltage. When n approaches infinity, U_n can be infinite. According to Electric Potential Relativity, if an electric potential limit exists, this is impossible; therefore, U_n is not a real voltage, but merely a theoretically calculated value.

However, U_0 is the measured voltage, which is equivalent to conducting the measurement of each identical battery in a zero-potential reference frame; thus, $U_0 = \Delta\phi_i = \Delta\phi_{i+1}$. It is worth noting that U_0 itself is a nonlinear potential difference, its magnitude is much smaller than the electric potential limit constant, and it must satisfy Eq. (117):

$$U_0 = \Delta\phi_i = \Phi_0 \tanh(\Delta\varphi) \tag{124}$$

For each battery, the voltage U_0 is also equal. Therefore, the corresponding hyperbolic function angle difference $\Delta\varphi$ for each battery is equal. Consequently, hyperbolic function angles can superpose linearly, and in a series circuit of n batteries, the following relationship holds:

$$\varphi = n\Delta\varphi \tag{125}$$

From Eqs. (123) (124) and (125) the relationship between U_n and φ is obtained:

$$\varphi = \frac{U_n}{\Phi_0 \tanh(\Delta\varphi)}\Delta\varphi \tag{126}$$

Eq. (126) is a discrete equation. As n approaches infinity, the voltage U_0 of a single battery approaches zero, and $\Delta\varphi$ approaches zero. Therefore, the linear superposition of the potential differences of infinitely many batteries and U_n approach a continuous linear electric potential ϕ_l, and the corresponding φ also becomes a continuous hyperbolic function angle. Utilizing $\lim\limits_{x\to 0}\frac{x}{\tanh x} = 1$, taking the limit of Eq. (126) yields a continuous functional relationship:

$$\varphi = \lim_{\Delta\varphi\to 0}\frac{U_n}{\Phi_0 \tanh(\Delta\varphi)}\Delta\varphi = \frac{\phi_\mathrm{l}}{\Phi_0} \tag{127}$$

Eq. (127) indicates that the physical significance of the hyperbolic function angle φ is the ratio of the linear potential ϕ_l to the limit potential constant Φ_0. From Eqs. (127) and (117), the relationship between the nonlinear potential ϕ and the linear potential ϕ_l is obtained:

$$\frac{\phi}{\Phi_0} = \tanh\left(\frac{\phi_\mathrm{l}}{\Phi_0}\right) \tag{128}$$

Eq. (128) establishes the global mapping relationship between the nonlinear physical potential ϕ and the mathematical linear potential ϕ_l (similar to the relationship between velocity and rapidity in Special Relativity). To investigate its local properties, taking the derivative of both sides of Eq. (128) with respect to ϕ_l yields:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\phi_\mathrm{l}} = 1 - \frac{\phi^2}{{\Phi_0}^2} \tag{129}$$

A fundamental physical contradiction emerges here: classical electrodynamics is built upon a global linear space, assuming that the electric potential ϕ can take arbitrary values and satisfies the linear superposition principle (i.e., global gauge invariance). However, Electrodynamic Spacetime Relativity points out that the physical potential is constrained by Φ_0, and space possesses intrinsic nonlinear curvature.

To strictly describe the relationship between nonlinear physical quantities and linear calculated quantities mathematically, the following three distinct electric potential concepts are defined herein:

1) Nonlinear potential (ϕ): It is a physically existent quantity strictly constrained by the electric potential limit constant Φ_0 ($|\phi| < \Phi_0$). Analogous to velocity in Special Relativity, it satisfies the nonlinear superposition principle.

2) Local linear potential (${\phi'}_\mathrm{l}$): It is the linear potential defined in the Local Tangent Space at the physical potential ϕ. As the effective linear potential (such as voltage) actually measured by a local observer, it satisfies the classical linear superposition principle; its value is modulated by a nonlinear factor, reflecting the "compression" effect of a high-potential background on the local metric.

3) Background linear potential (ϕ_l): It is the local linear potential defined where the physical potential is zero ($\phi = 0$), corresponding exactly to the theoretical value calculated by classical electrodynamics (such as the Coulomb potential). It is the "intrinsic" reference value unaffected by nonlinear spacetime curvature, mathematically constituting an abstract background space that satisfies the global linear superposition principle. It is an invariant in any equipotential reference frame.

Based on the principles of differential geometry, the observable local linear potential differential in the Local Tangent Space is strictly defined as the differential of the nonlinear potential. Namely:

$$\mathrm{d}\phi_{\mathrm{l}}' = \mathrm{d}\phi \tag{130}$$

Simultaneously, according to the global mapping relationship established by Eq. (129), taking the derivative with respect to the background linear potential ϕ_{l} reveals a deterministic transformation relationship between its differential and the physical potential differential. Combining this with Eq. (130), the transformation relationship between the local linear potential differential $\mathrm{d}\phi'_{\mathrm{l}}$ and the background linear potential differential $\mathrm{d}\phi_{\mathrm{l}}$ is obtained:

$$\frac{\mathrm{d}\phi_{\mathrm{l}}'}{\mathrm{d}\phi_{\mathrm{l}}} = \frac{\mathrm{d}\phi}{\mathrm{d}\phi_{\mathrm{l}}} = 1 - \frac{\phi^2}{\Phi_0^2} \tag{131}$$

Because the electric potential Lorentz factor in Electric Potential Relativity is $\gamma(\phi) = \frac{1}{\sqrt{1-(\phi/\Phi_0)^2}}$, therefore,

$$\gamma^{-2}(\phi) = 1 - \frac{\phi^2}{{\Phi_0}^2} \tag{132}$$

Since ϕ'_{l} and ϕ_{l} are defined as linear physical quantities (satisfying the linear superposition principle) in the local tangent space and the background space, respectively, and ϕ is invariant within an equipotential reference frame, $\gamma(\phi)$ can be treated as a constant. The differential proportionality relationship in Eq. (131) is thus directly extended to the linear potentials themselves, yielding:

$$\phi_{\mathrm{l}}' = \gamma^{-2}(\phi)\phi_{\mathrm{l}} \tag{133}$$

Eq. (133) reveals extremely profound physical connotations; it indicates that the nonlinear nature of the potential reference frame leads to a contraction effect of the local metric:

First, it modifies the classical picture. The linear potential ϕ_{l} in classical electrodynamics is "compressed" by the nonlinear factor $\gamma^{-2}(\phi)$ into the local linear potential ϕ'_{l} under a high-potential ϕ background. This implies that in high-energy environments, the effective coupling strength of the electromagnetic field undergoes changes.

Second, it ensures the self-consistency and continuity of the theory. When the background potential is much smaller than the limit constant $\phi \ll \Phi_0$, the nonlinear factor $\gamma^{-2}(\phi) \to 1$, and at this time $\phi'_{\mathrm{l}} \approx \phi_{\mathrm{l}}$. This demonstrates that at the low-energy limit, the local tangent space tends to coincide with the global space of classical electrodynamics. The present theory naturally degenerates into the classical linear Maxwell electromagnetic theory; satisfying this "correspondence principle" is an important criterion for verifying the physical reality of a new theory.

Finally, it provides the foundation for the reconstruction of gauge symmetry. The classical Maxwell's equations rely on the gauge invariance of linear space. By introducing the local linear potential ϕ'_{l}, a local flat space is successfully constructed at every nonlinear potential point, enabling the generalization of classical differential operators to nonlinear manifolds, thereby providing a strict mathematical and physical foundation for the subsequent derivation of the nonlinear Maxwell's equations.

Based on the aforementioned physical picture, the specific differential transformation relationships can be formulated. From Eq. (133), the relationship between the linear potential difference (i.e., voltage) and the reference frame potential is known. Let $U = \Delta\phi_{\mathrm{l}}$ be the background linear voltage, and $U' = \Delta\phi'_{\mathrm{l}}$ be the local linear voltage; thus:

$$U' = \gamma^{-2}(\phi)U \tag{134}$$

Obviously, when $\phi \ll \Phi_0, \gamma^{-2}(\phi) \approx 1$, and Eq. (134) approximately simplifies to $U' \approx U$. This mathematically corroborates once again that classical electrodynamics is the low-energy approximation of this theory.

Currently, all necessary conditions for modifying Maxwell's equations have been met. Next, the nonlinear Maxwell's equations will be specifically derived.

In a vacuum, the classical Maxwell's equations take the following integral forms:

$$\oiint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = \frac{q}{\varepsilon_0} \tag{135}$$

$$\oiint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = 0 \tag{136}$$

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{s} \tag{137}$$

$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 I + \mu_0 \varepsilon_0 \iint \frac{\partial \boldsymbol{E}}{\partial t} \cdot \mathrm{d}\boldsymbol{s} \tag{138}$$

According to the principle of relativity in Electric Potential Relativity, physical laws share the same form in any equipotential reference frame. Therefore, in a reference frame with an electric potential of ϕ, Maxwell's equations take the following forms:

$$\oiint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{s}' = \frac{q'}{\varepsilon_0'} \tag{139}$$

$$\oiint \boldsymbol{B}' \cdot \mathrm{d}\boldsymbol{s}' = 0 \tag{140}$$

$$\oint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{l}' = -\iint \frac{\partial \boldsymbol{B}'}{\partial t'} \cdot \mathrm{d}\boldsymbol{s}' \tag{141}$$

$$\oint \boldsymbol{B}' \cdot \mathrm{d}\boldsymbol{l}' = \mu_0' I' + \varepsilon_0' \mu_0' \iint \frac{\partial \boldsymbol{E}'}{\partial t'} \cdot \mathrm{d}\boldsymbol{s}' \tag{142}$$

In classical electrodynamics, the physical quantities in the aforementioned Maxwell's equations are invariants in any stationary equipotential reference frame. In Electric Potential Relativity, although the forms of these two sets of equations are identical, some of the physical quantities within them may be invariants, while others may be related to the reference frame potential ϕ.

3.1.1 Modification of Gauss's Law for the Electric Field

According to Eq. (120), the lengths of three-dimensional space are independent of the magnitude of the reference frame's electric potential, namely:

$$\mathrm{d}r' = \mathrm{d}r \tag{143}$$

The definition of a spherically symmetric electric field is: $\boldsymbol{E}' = -\frac{\mathrm{d}\phi}{\mathrm{d}r'} = -\frac{d\phi_1'}{dr'}$, and $\boldsymbol{E} = -\frac{\mathrm{d}\phi_1}{\mathrm{d}r}$. From Eqs. (129), (132), and(143), it follows that:

$$\boldsymbol{E}' = \gamma^{-2}(\phi)\boldsymbol{E} \tag{144}$$

Because the three-dimensional geometric dimensions are independent of the electric potential, namely:

$$\mathrm{d}\boldsymbol{s} = \mathrm{d}\boldsymbol{s}' \tag{145}$$

Therefore, by integrating both sides of Eq. (144) simultaneously, Gauss's law is obtained as:

$$\oiint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{s}' = \gamma^{-2}(\phi) \oiint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} \tag{146}$$

From Eqs. (145), (135), and (146), the modified Gauss's law for the electric field is obtained:

$$\oiint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{s}' = \gamma^{-2}(\phi) \frac{q}{\varepsilon_0} \tag{147}$$

This indicates that in nonlinear spacetime, the actually observed electric field intensity $\boldsymbol{E}'$ is "weaker" than the $\boldsymbol{E}$ predicted by linear theory (because $\gamma^{-2}(\phi) < 1$). This can be understood as a "dielectric shielding" effect occurring in the vacuum at high electric potentials.

Comparing Eqs. (139) and (147) yield:

$$\frac{q'}{\varepsilon_0'} = \gamma^{-2}(\phi) \frac{q}{\varepsilon_0} \tag{148}$$

To prove that the electric charge is independent of the reference frame's electric potential $(q' = q)$, the behavior of the law of charge conservation under equipotential transformations is examined. According to the classical Maxwell's equations, the charge continuity equation in a reference frame with zero potential is given by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{V} = 0 \tag{149}$$

According to the principle of relativity, the charge continuity equation should maintain form invariance in any equipotential reference frame Σ'_ϕ:

$$\frac{\partial\rho'}{\partial t'} + \nabla' \cdot \rho' \boldsymbol{V}' = 0 \tag{150}$$

Substituting the previously derived spacetime and velocity transformation relations $t' = \gamma(\phi)\ t$, $\boldsymbol{V}' = \gamma^{-1}(\phi)\boldsymbol{V}$, and $\nabla' = \nabla$ nto the above equation yields:

$$\gamma^{-1}(\phi)\left(\frac{\partial\rho'}{\partial t} + \nabla \cdot \rho' \boldsymbol{V}\right) = 0 \tag{151}$$

The above equation indicates that, mathematically, it is permissible for the charge density ρ' to undergo some proportional scaling with the electric potential (i.e., $\rho' = \mathrm{f}(\phi)\rho$) without violating the covariance of the equation. However, physical reality imposes strict constraints on $\mathrm{f}(\phi)$. If the charge varied with the electric potential ($\mathrm{f}(\phi) \neq 1$), then in a multi-electron atomic system, electrons at different potential energy levels would carry different charges, which would disrupt the overall electrical neutrality of the atom. Extensive experimental evidence demonstrates that atoms strictly maintain electrical neutrality in various environments, compelling the theory to adopt $\mathrm{f}(\phi) \equiv 1$.

Therefore, the conclusion is reached: in Electrodynamic Spacetime Relativity, the charge density and the electric charge are potential invariants, namely:

$$\rho' = \rho \tag{152}$$

$$q' = q \tag{153}$$

This proves that the magnitude of the charge is not only independent of the observer's state of motion (the classical relativistic conclusion) but also independent of the magnitude of the electric potential where the observer is located.

From Eqs. (148) and (153), it follows that:

$$\varepsilon_0' = \gamma^2(\phi)\varepsilon_0 \tag{154}$$

Eq. (154) reveals the singular behavior of the vacuum dielectric properties at the limit electric potential: as $\phi \to \Phi_0$, the electric potential Lorentz factor $\gamma(\phi) \to \infty$, leading to $\varepsilon_0' \to \infty$.

From a physical perspective, this implies that the vacuum exhibits an extremely strong dielectric response (or "hyper-polarization" effect) as it approaches the electric potential limit. This intrinsically strong shielding mechanism can effectively suppress the growth of the electric field intensity, thereby circumventing the problem of the point charge electric field energy diverging to infinity in classical electrodynamics. This is a field-strength truncation mechanism induced by the nonlinearity of spacetime geometry; its physical picture shares a profound qualitative consistency with the vacuum polarization effect in Quantum Electrodynamics (QED), implying that the vacuum is not entirely empty, but participates in interactions and modifies the effective parameters of the physical fields.

3.1.2 Modification of Gauss's Law for Magnetism

Suppose that in a reference frame with zero electric potential, the magnetic induction intensity inside an infinitely long solenoid is $\boldsymbol{B}$, and in a reference frame with an electric potential of ϕ, the magnetic induction intensity inside the same solenoid is $\boldsymbol{B}'$, thus yielding:

$$\boldsymbol{B} = \mu_0 n \boldsymbol{I} \tag{155}$$

$$\boldsymbol{B}' = \mu_0' n' \boldsymbol{I}' \tag{156}$$

Here, $\mu_0, n,\ \boldsymbol{I}$, and $\mu'_0, n',\ \boldsymbol{I}'$ are respectively the vacuum permeability, the number of turns of the coil, and the current intensity vector in the reference frame with zero potential and the reference frame with potential ϕ. Because geometric parameters are independent of the electric potential, $n' = n$, and $\boldsymbol{B}'$ and $\boldsymbol{B}$ are magnetic induction intensity vectors in the same direction.

$$\frac{\boldsymbol{B}'}{\boldsymbol{B}} = \frac{\mu_0' \boldsymbol{I}'}{\mu_0 \boldsymbol{I}} \tag{157}$$

According to the definition of current, from Eqs. (111), (148), and (154), it follows that:

$$\frac{I'}{I} = \gamma^{-3}(\phi)\frac{\varepsilon_0'}{\varepsilon_0} = \gamma^{-1}(\phi) \tag{158}$$

From Eqs. (158) and (157), it is obtained that:

$$\boldsymbol{B}' = \gamma^{-3}(\phi)\frac{\varepsilon_0'\mu_0'}{\varepsilon_0\mu_0}\boldsymbol{B} = \gamma^{-1}(\phi)\frac{\mu_0'}{\mu_0}\boldsymbol{B} \tag{159}$$

From Eqs. (136), (145), and (159), it follows that:

$$\oiint \boldsymbol{B}' \cdot \mathrm{d}\boldsymbol{s}' = \gamma^{-1}(\phi)\frac{\mu_0'}{\mu_0}\oiint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = 0 \tag{160}$$

Therefore, after comparing Eq. (160) with Eq. (140), which comes directly from the principle of potential relativity, they are exactly the same. This indicates that in nonlinear electromagnetic theory, magnetic monopoles still do not exist.

3.1.3 Modification of Faraday's Law of Induction

Substituting Eqs. (159), (111), and (145) into Eq. (141) yields:

$$\oint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{l}' = -\gamma^{-4}(\phi)\frac{\varepsilon_0'\mu_0'}{\varepsilon_0\mu_0}\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{s} \tag{161}$$

Substituting Eq. (137) into (161) yields:

$$\oint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{l}' = \gamma^{-4}(\phi)\ \frac{\varepsilon_0'\mu_0'}{\varepsilon_0\mu_0}\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} \tag{162}$$

Because three-dimensional geometric dimensions are independent of the electric potential, thus $\mathrm{d}\boldsymbol{l}' = \ d\boldsymbol{l}$. From Eq. (144), it follows that:

$$\oint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{l}' = \gamma^{-2}(\phi)\ \oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} \tag{163}$$

Comparing Eqs. (162) and (163) yield:

$$\frac{\varepsilon_0'\mu_0'}{\varepsilon_0\mu_0} = \gamma^{2}(\phi) \tag{164}$$

Substituting the above equation into Eq. (159) yields:

$$\frac{\boldsymbol{B}'}{\boldsymbol{B}} = \gamma^{-1}(\phi) \tag{165}$$

It is evident that the magnetic induction intensity is related to the reference frame's electric potential. Substituting Eq. (154) into Eq. (164) yields:

$$\mu_0' = \mu_0 \tag{166}$$

Substituting Eq. (164) into Eq. (161) yields the modified Faraday's law of induction:

$$\oint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{l}' = -\gamma^{-2}(\phi)\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot \mathrm{d}\boldsymbol{s} \tag{167}$$

3.1.4 Modification of the Maxwell-Ampere Circuital Law

From Eqs. (157) and (165), it follows that**:**

$$\frac{\mu_0' I'}{\mu_0 I} = \frac{I'}{I} = \gamma^{-1}(\phi) \tag{168}$$

Let $\boldsymbol{J}$ be the current density in a reference frame with zero electric potential, and $\boldsymbol{J}'$ be the current density in a reference frame with an electric potential of ϕ. Because volume is independent of the reference frame's electric potential, from Eq. (168) it follows that:

$$\boldsymbol{J}' = \gamma^{-1}(\phi)\boldsymbol{J} \tag{169}$$

Substituting Eqs. (168), (144), (111), and (164) into Eq. (142), and utilizing $\mathrm{d}\boldsymbol{s} = \mathrm{d}\boldsymbol{s}'$**,** the modified Maxwell-Ampere circuital law is obtained:

$$\oint \boldsymbol{B}' \cdot \mathrm{d}\boldsymbol{l}' = \gamma^{-1}(\phi)\left(\mu_0 I + \mu_0\varepsilon_0 \iint \frac{\partial \boldsymbol{E}}{\partial t} \cdot \mathrm{d}\boldsymbol{s}\right) \tag{170}$$

3.1.5 Relationship between the Covariance of the Speed of Light and the Principle of the Constancy of the Speed of Light

With the establishment of the nonlinear Maxwell's equations, particularly the determination of the permittivity transformation relation (154) and the permeability invariance (166), the definition of the speed of light can now be re-examined from an electrodynamic perspective.

In classical electrodynamics and Special Relativity, the speed of light in a vacuum is

$C_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$, which is the inherent speed of light when the reference frame's electric potential is zero. However, in a reference frame with an electric potential of ϕ, the coordinate speed of light C_0' for electromagnetic waves is determined by local dielectric parameters. According to Eq. (164), it follows that:

$$C_0' = \frac{1}{\sqrt{\varepsilon_0'\mu_0'}} = \frac{1}{\sqrt{\gamma^2(\phi)\varepsilon_0\mu_0}} = \gamma^{-1}(\phi)\, C_0 \tag{171}$$

This equation also elucidates the physical significance of the nonlinear potential factor in Eq. (132) :

$$\gamma^{-1}(\phi) = \frac{C_0'}{C_0} \tag{172}$$

According to Complex Electrodynamic Spacetime Relativity, this electric potential ϕ can be converted into an imaginary velocity state iV_ϕ, and the equation $V_\phi = C_0\frac{\phi}{\Phi_0}$ holds. Therefore, Eq. (171) can be rewritten into a more profound Pythagorean form:

$${C_0'}^2 + {C_0}^2\frac{\phi^2}{{\Phi_0}^2} = {C_0'}^2 + {V_\phi}^2 = {C_0}^2 \tag{173}$$

This conclusion is completely identical to Eq. (26), which is a corollary of the light speed invariance postulate (A6) of Complex Electrodynamic Spacetime Relativity. It reveals the deep geometric mechanism of the principle of the constancy of the speed of light: in complex electrodynamic spacetime, the complex velocity modulus C_0 of a photon in a complex electrodynamic inertial reference frame is an invariant, meaning the inherent speed of light C_0 is independent of the reference frame's state**.** Although the value of the inherent speed of light C_0 remains unchanged in high-potential space, it undergoes a "rotation." Specifically, the component of the inherent speed of light C_0 in the imaginary dimension (potential dimension) is iV_ϕ, and the component in real three-dimensional space is C_0' (i.e., the coordinate speed of light, which is the limit value of real motion velocity in a reference frame with a nonlinear electric potential of ϕ). When the nonlinear electric potential ϕ of the reference frame is zero, $C_0' = C_0$, and the principle of the constancy of the speed of light in Complex Electrodynamic Spacetime Relativity reverts to the principle of the constancy of the speed of light in Special Relativity.

3.1.6 Relationship Between Physical Quantities and Reference Frame Potential in Nonlinear Electromagnetic Theory

Based on the derivations of Electric Potential Relativity and the nonlinear Maxwell's equations, the relationship between fundamental physical quantities and the reference frame electric potential can be obtained. Through the definitions or relationships between these fundamental physical quantities and other physical quantities, the relationships of additional physical quantities with the reference frame electric potential can be derived via simple algebraic operations. To effectively compare these with the identical physical quantities in classical electrodynamics, Table 1 is formulated as follows:

Table 1: Comparison table of physical parameters in electromagnetic theory

Content	Classical Electrodynamics	Non-linear Electrodynamics
Nonlinear Potential*	$\phi = 0$	$\lvert\phi\rvert < \Phi_0$
Nonlinear Electric Potential Lorentz Factor	$\gamma(\phi) = 1$	$\gamma(\phi) = \left(1 - \frac{\phi^2}{{\Phi_0}^2}\right)^{-\frac{1}{2}}$
Space	x, y, z, π	$x' = x, y' = y, z' = z, \pi' = \pi$

Time	t	$t' = \gamma(\phi)t$
Spacetime Operators	$\nabla, \quad \dfrac{\partial}{\partial t}$	$\nabla' = \nabla, \frac{\partial}{\partial t'} = \gamma^{-1}(\phi)\frac{\partial}{\partial t}$
Proper Speed of Light*	C_0	C_0
Coordinate Speed of Light*	C_0	$C_0' = \gamma^{-1}(\phi)C_0$
Velocity and Acceleration	$\boldsymbol{v}, \ \boldsymbol{a}$	$\boldsymbol{v}' = \gamma^{-1}(\phi)\boldsymbol{v}, \boldsymbol{a}' = \gamma^{-2}(\phi)\boldsymbol{a}$
Electric Charge	q	$q' = q$
Charge Density	ρ	$\rho' = \rho$
Local Linear Potential*	ϕ_l'	$\phi_l' = \gamma^{-2}(\phi)\phi_l$
Background Linear Potential*	ϕ_l	ϕ_l
Linear Potential Difference (Voltage)	$U = \Delta\phi_l$	$U' = \gamma^{-2}(\phi)U = \Delta\phi_l'$
Vacuum Permittivity	ε_0	$\varepsilon_0' = \gamma^2(\phi)\varepsilon_0$
Vacuum Permeability	μ_0	$\mu_0' = \mu_0$
Electric Field Intensity	$\boldsymbol{E}$	$\boldsymbol{E}' = \gamma^{-2}(\phi)\boldsymbol{E}$
Magnetic Flux Density	$\boldsymbol{B}$	$\boldsymbol{B}' = \gamma^{-1}(\phi)\boldsymbol{B}$
Magnetic Vector Potential	$\boldsymbol{A}$	$\boldsymbol{A}' = \gamma^{-1}(\phi)\boldsymbol{A}$
Electric Current	I	$\mathrm{I}' = \gamma^{-1}(\phi)\mathrm{I}$
Current Density	$\boldsymbol{J}$	$\boldsymbol{J}' = \gamma^{-1}(\phi)\boldsymbol{J}$
Lagrangian Density	$\mathcal{L}$	$\mathcal{L}' = \gamma^{-2}(\phi)\mathcal{L}$
Resistance, Capacitance, Inductance	$R_\mathrm{e}, C_\mathrm{e}, L_\mathrm{e}$	$R_e' = \gamma^{-1}(\phi)R_e,$ $C_e' = \gamma^2(\phi)C_e, L_e' = L_e$
Frequency	ν	$\nu' = \gamma^{-1}(\phi)\nu$
Planck Constant, Angular Momentum	$h, \ \boldsymbol{L}$	$h' = \gamma^{-1}(\phi)h, \boldsymbol{L}' = \gamma^{-1}(\phi)\boldsymbol{L}$
Fine-structure Constant	α	$\alpha' = \alpha$
Energy and Momentum	$E_\mathrm{e}, \ \mathbf{p}$	$E_\mathrm{e}' = \gamma^{-2}(\phi)E_\mathrm{e}, \boldsymbol{p}' = \gamma^{-1}(\phi)\boldsymbol{p}$
Mass and Force	$m, \boldsymbol{F}$	$m' = m, \quad \boldsymbol{F}' = \gamma^{-2}(\phi)\boldsymbol{F}$
Gravitational Constant	G	$G' = \gamma^{-2}(\phi)G$
Energy-Momentum Tensor	$T^{\mu\nu}$	$T'^{\mu\nu} = \gamma^{-2}(\phi)T^{\mu\nu}$

Footnotes:

* For definitions of Nonlinear Potential ϕ, Local Linear Potential ϕ_l', and Background Linear Potential ϕ_l, please refer to Section 3.1 *Nonlinear Maxwell's Equations*.

* Proper Speed of Light: The speed of light in a vacuum within a reference frame where the potential is zero.

* Coordinate Speed of Light: The speed of light in a reference frame with a non-zero potential.

* In the same equation, primed quantities ($'$) represent physical quantities in a local reference frame with potential, while unprimed quantities represent those in a zero-potential reference frame.

3.2 Unified Expression Forms of Nonlinear Maxwell's Equations

According to the principle of relativity within Electric Potential Relativity, this theory maintains strict form invariance of the classical Maxwell's equations within a local reference frame. To simultaneously reveal this covariance and the specific modulation mechanism of the nonlinear electric potential Lorentz factor $\gamma(\phi)$, this section adopts a continuous equality format. Specifically, through the structure of "local covariant form = classical expanded form containing the $\gamma(\phi)$ factor = local covariant source term," it intuitively demonstrates how the nonlinear factor is introduced in the definitions of physical quantities and exactly canceled out on both sides of the equations, thereby preserving the classical mathematical structure of Maxwell's equations.

3.2.1 Integral Forms

By organizing the derived nonlinear Maxwell's equations, namely Eqs. (147), (160), (167), and (170) in conjunction with the transformation relations of relevant physical quantities in Table 1, the following is obtained:

$$\oiint \boldsymbol{E}' \cdot d\boldsymbol{s}' = \gamma^{-2}(\phi) \oiint \boldsymbol{E} \cdot d\boldsymbol{s} = \frac{q'}{\varepsilon_0'} \tag{174}$$

$$\oiint \boldsymbol{B}' \cdot d\boldsymbol{s}' = \gamma^{-1}(\phi) \oiint \boldsymbol{B} \cdot d\boldsymbol{s} = 0 \tag{175}$$

$$\oint \boldsymbol{E}' \cdot d\boldsymbol{l}' = \gamma^{-2}(\phi) \oint \boldsymbol{E} \cdot d\boldsymbol{l} = -\iint \frac{\partial \boldsymbol{B}'}{\partial t'} \cdot d\boldsymbol{s}' \tag{176}$$

$$\oint \boldsymbol{B}' \cdot d\boldsymbol{l}' = \gamma^{-1}(\phi) \oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0' I' + \varepsilon_0' \mu_0' \iint \frac{\partial \boldsymbol{E}'}{\partial t'} \cdot d\boldsymbol{s}' \tag{177}$$

3.2.2 Differential Forms

Utilizing Gauss's Divergence Theorem and Stokes' Theorem, the aforementioned integral equations can be transformed into differential forms that describe the local field behavior at every point in space:

$$\boldsymbol{\nabla}' \cdot \boldsymbol{E}' = \gamma^{-2}(\phi) \boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho'}{\varepsilon_0'} \tag{178}$$

$$\boldsymbol{\nabla}' \cdot \boldsymbol{B}' = \gamma^{-1}(\phi) \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{179}$$

$$\boldsymbol{\nabla}' \times \boldsymbol{E}' = \gamma^{-2}(\phi) \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}'}{\partial t'} \tag{180}$$

$$\boldsymbol{\nabla}' \times \boldsymbol{B}' = \gamma^{-1}(\phi) \boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0' \boldsymbol{J}' + \varepsilon_0' \mu_0' \frac{\partial \boldsymbol{E}'}{\partial t'} \tag{181}$$

3.2.3 Tensor Forms

Similarly, based on the fact that the tensor forms of the classical Maxwell's equations and the nonlinear Maxwell's equations are identical, along with the definitions and transformation relations of the relevant physical quantities, the tensor forms of the nonlinear Maxwell's equations are obtained:

$$\partial_\nu' F'^{\mu\nu} = \gamma^{-1}(\phi) \partial_\nu F^{\mu\nu} = \mu_0' \boldsymbol{J}'^{\mu} \tag{182}$$

$$\partial_\lambda' F_{\mu\nu}' + \partial_\mu' F_{\nu\lambda}' + \partial_\nu' F_{\lambda\mu}' = \gamma^{-1}(\phi) \left(\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} \right) = 0 \tag{183}$$

3.3 Generalized Gauge Invariance and Conservation Laws

The formal unification of the aforementioned nonlinear Maxwell's equations implies a deep underlying dynamical symmetry. To verify this, the electromagnetic Lagrangian density $\mathcal{L}'$ in the local reference frame is examined.

The "linear potential" ϕ'_1 (the effective potential satisfying the linear superposition law) defined previously, and the vector potential $\boldsymbol{A}'$ are introduced into the interaction term to construct the local Lagrangian density. After substituting the transformation relations of physical quantities, it is found that a strict geometric scaling relationship exists between it and the classical Lagrangian density $\mathcal{L}$:

$$\mathcal{L}' = \frac{1}{2} \varepsilon_0' E'^2 - \frac{1}{2\mu_0'} B'^2 - \rho' \phi_1' + \boldsymbol{J}' \cdot \boldsymbol{A}' = \gamma^{-2}(\phi) \mathcal{L} \tag{184}$$

This result indicates that the electric potential Lorentz factor $\gamma(\phi)$ appears merely as a global conformal factor and does not alter the internal algebraic structure of the Lagrangian. Based on this, the following two key conclusions are drawn:

1) Generalized Gauge Invariance

Nonlinear electromagnetic theory satisfies generalized gauge invariance: that is, within the local linear reference frame of any tangent space, the gauge transformation of the electromagnetic potential does not change the physical essence, and the system possesses complete gauge degrees of freedom; the nonlinear effect is manifested only as a global conformal scaling of the Lagrangian density between different reference frames. Under this framework, classical gauge invariance is simply a special case of this theory under the zero-potential condition ($\gamma(\phi) = 1, \phi'_1 = \phi_1$). This indicates that the U(1) gauge symmetry [14] of classical electrodynamics is completely preserved within the local structure of this theory.

2) Verification of Charge Conservation under Generalized Gauge Invariance

According to Noether's theorem, every continuous symmetry of the Lagrangian corresponds to a conserved current. Since this theory satisfies generalized gauge invariance, this requires, at the level of dynamical principles, the existence of strict charge conservation. In the local reference frame, the divergence of the four-dimensional current density is strictly zero:

$$\frac{\partial \rho'}{\partial t'} + \boldsymbol{\nabla}' \cdot \boldsymbol{J}' = 0 \tag{185}$$

To verify the consistency between this theoretical result and the conclusion derived earlier (Section 3.1.1) based on physical observation (atomic electrical neutrality), the transformation relations $t' = \gamma(\phi)\ t, \boldsymbol{\nabla}' = \boldsymbol{\nabla}, \rho' = \rho$, and $\boldsymbol{J}' = \gamma^{-1}(\phi)\,\boldsymbol{J}$ are substituted into the above equation.

$$\gamma^{-1}(\phi)\left(\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J}\right) = 0 \tag{186}$$

Because $\gamma^{-1}(\phi) \neq 0$, the above equation implies that the classical continuity equation $\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{J} = 0$ still holds.

This derivation possesses significant theoretical implications: it demonstrates that "charge conservation" and the "invariance of charge quantity q" are not externally introduced assumptions (such as the experimental constraints mentioned earlier), but are inevitable mathematical corollaries of the generalized gauge symmetry within Electrodynamic Spacetime Relativity. This not only proves the high degree of internal self-consistency of the theory but also reveals that the introduction of a nonlinear factor $\gamma^{-1}(\phi)$ does not disrupt the conservation foundation of electromagnetic interactions.

3) Energy and Momentum Conservation under Generalized Gauge Invariance

Similarly, because the local Lagrangian density $\mathcal{L}'$ maintains spacetime translational invariance (i.e., time homogeneity and spatial homogeneity) within the local reference frame, according to Noether's theorem, the energy-momentum conservation law $\partial'_\mu T'^{\mu\nu} = 0$ is also strictly valid in the local reference frame. This indicates that the introduction of a nonlinear factor does not disrupt the fundamental spacetime conservation upon which the physical system relies; energy and momentum still follow classical conservation and transformation processes during local interactions.

Therefore, this theory strictly satisfies generalized gauge invariance. The concept of linear potential plays a crucial bridging role here; it ensures that against the background of a nonlinear reference frame potential ϕ, local electromagnetic interactions still follow classical superposition principles and conservation laws.

In summary, nonlinear electromagnetic theory essentially constructs a geometric framework that is "globally nonlinear but locally linear." Every nonlinear potential reference frame ϕ corresponds to a local tangent space (linear space) on the manifold, where the classical linear Maxwell's equations remain valid within this tangent space, and the linear Maxwell's equations between tangent spaces differ only by a non-zero constant factor. This is equivalent to expressing the nonlinear Maxwell's equations as an ensemble of linear Maxwell's equations across infinitely many tangent spaces.

Through this geometric reconstruction, despite the introduction of the electric potential Lorentz factor $\gamma(\phi)$, it is proven that the modified nonlinear Maxwell's equations maintain strict covariance in integral, differential, and tensor forms, and possess generalized gauge transformation invariance. This theory perfectly inherits the U(1) gauge symmetry of electromagnetic interactions and serves as its natural generalization on nonlinear spacetime manifolds. Crucially, through the nonlinear renormalization mechanism of vacuum dielectric parameters, this theory fundamentally eliminates the point charge divergence (singularity) problem in classical electrodynamics, laying a solid foundation for the subsequent theoretical predictions and experimental verifications of new physical effects.

3.4 New Physical Effects

Based on the modified Maxwell's equations, Parameter Comparison Table 1 enumerates the relationships between multiple physical quantities and the reference frame electric potential. Each relationship corresponds to a physical effect (including invariants). While it is impossible to discuss them all here, several representative physical effects will be listed and briefly discussed below.

3.4.1 Electric Potential Refraction Effect and Electric Potential Lensing Effect

Because the electric potential induces a change in the speed of light in three-dimensional space, if Fermat's principle remains valid—that is, the law of refraction still holds—it indicates that when light passes through a space with a potential difference, an electric potential refraction effect exists:

Suppose two adjacent spaces are S_1 and S_2, and ϕ_1, C_1', and n_1 denote the electric potential, the speed of light, and the refractive index in space S_1 respectively, while ϕ_2, C_2', and n_2 denote the electric potential, the speed of light, and the refractive index in space S_2 respectively. From Eq. (171), it follows that:

$$n_1 = \frac{C_0}{C_1'} = \gamma(\phi_1) \tag{187}$$

$$n_2 = \frac{C_0}{C_2'} = \gamma(\phi_2) \tag{188}$$

When a light ray propagates from space S_1 into space S_2, the law of refraction is expressed by the equation:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{189}$$

Where θ_1 is the angle of incidence, and θ_2 is the angle of refraction.

Consider a distant light source emitting a light beam that passes near a charged spherical body. The closer the light beam is to the spherical body, the greater the absolute value of the electric potential in the space where the light beam is located. According to the definition of $\gamma(\phi)$, from Eqs. (188) and (187), it is known that:

$$n_2 > n_1 \tag{190}$$

From Eq. (189), it is known that:

$$\theta_1 > \theta_2 \tag{191}$$

Therefore, when a light ray passes near a charged spherical body, its path bends towards the center of the body; this is termed the electric potential lensing effect. From the refractive index equation, it is known that the refractive index depends on the square of ϕ. This implies that regardless of whether the central charged body carries a positive or negative charge, the resulting refractive index is always greater than 1. They produce a converging (convex lens) effect on light. Formally, the electric potential lensing effect is similar to the gravitational lensing effect caused by mass, but the origins of these lensing effects are different.

3.4.2 Electric Potential Redshift Effect

Consider an $L_\mathrm{e}C_\mathrm{e}$ electromagnetic oscillation circuit located in a reference frame with zero electric potential. The circuit consists of an inductor L_e and a capacitor C_e, and its inherent electromagnetic oscillation frequency is ν. When this circuit is placed in a reference frame with an electric potential of ϕ, its physical parameters undergo transformations: the inductance becomes L_e', and the capacitance becomes C_e'. At this point, the measured electromagnetic oscillation frequency of the circuit is ν'.

$$\nu = \frac{1}{2\pi\sqrt{L_{\mathrm{e}}C_{\mathrm{e}}}} \tag{192}$$

$$\nu' = \frac{1}{2\pi'\sqrt{L'_{\mathrm{e}}C'_{\mathrm{e}}}} \tag{193}$$

Because geometric dimensions are independent of the reference frame's electric potential, $\pi = \pi'$.

Substituting the relationships between capacitance and inductance and the reference frame electric potential listed in Parameter Comparison Table 1 ($L'_{\mathrm{e}} = L_{\mathrm{e}}$, $C'_{\mathrm{e}} = \gamma^2(\phi)C_{\mathrm{e}}$) into Eq. (193) yields:

$$\nu' = \gamma^{-1}(\phi)\nu \tag{194}$$

Eq. (194) shows that the frequency of an electromagnetic wave is related to the reference frame's electric potential. Moreover, regardless of whether the observed electric potential of the reference frame is positive or negative, as long as $\phi \neq 0$, the relationship $\nu' < \nu$ always holds. That is, the frequency ν' of an electromagnetic wave in a reference frame with an electric potential of ϕ is always lower than the frequency ν of an electromagnetic wave in a reference frame with zero electric potential. This is referred to herein as the electric potential redshift effect. Since light is also an electromagnetic wave, light also exhibits an electric potential redshift effect. This effect of light may help explain significant astrophysical phenomena in cosmology.

3.4.3 Electric Potential Time Dilation Effect

Consider an $R_{\mathrm{e}}C_{\mathrm{e}}$ series circuit located in a reference frame with zero electric potential, composed of a resistor R_{e} and a capacitor C_{e}, with a characteristic time constant of τ. When this circuit is placed in a reference frame with an electric potential of ϕ, its physical parameters undergo transformations: the resistance becomes R'_{e}, and the capacitance becomes C'_{e}. At this point, the measured time constant of this $R'_{\mathrm{e}}C'_{\mathrm{e}}$ circuit is τ', meaning:

$$R_{\mathrm{e}}C_{\mathrm{e}} = \tau \tag{195}$$

$$R'_{\mathrm{e}}C'_{\mathrm{e}} = \tau' \tag{196}$$

Substituting the relationships between capacitance and resistance and the reference frame electric potential listed in Table 1 ($R'_{\mathrm{e}} = \gamma^{-1}(\phi)R_{\mathrm{e}}$, $C'_{\mathrm{e}} = \gamma^2(\phi)C_{\mathrm{e}}$) into Eq. (196) yields:

$$\tau' = \gamma(\phi)\,\tau \tag{197}$$

Eq. (111) is a corollary of Electric Potential Relativity, and Eq. (197) is a corollary of the modified Maxwell's equations. Comparing them reveals that they are mathematically equivalent.

3.4.4 Relationship Between Electric Potential and the Planck Constant

From Parameter Comparison Table 1, the following relationship exists between the Planck constant h' and h:

$$h' = \gamma^{-1}(\phi)h \tag{198}$$

Eq. (198) indicates that the Planck constant h is not an invariant scalar constant. It is precisely because h undergoes a corresponding change along with the spacetime geometry $\gamma^{-1}(\phi)$ that it offsets the changes in ε'_0 and C'_0, keeping the dimensionless fine-structure constant invariant (discussed in the next section). Simultaneously, it is noted that as ϕ approaches the electric potential limit constant Φ_0, both $\gamma^{-1}(\phi)$ and h' approach zero, meaning that quantum physical phenomena will vanish at the center of a point charge.

3.4.5 The Fine-Structure Constant and the Stability of Atomic Scales

By substituting the relevant physical quantities from Parameter Comparison Table 1 into the definition equation of the fine-structure constant, it is found that the nonlinear factor $\gamma(\phi)$ carried by each physical quantity cancels out completely:

$$\alpha' = \frac{q'^2}{4\pi\varepsilon'_0\hbar' C'_0} = \alpha \tag{199}$$

Similarly, when substituting the physical quantities into the definition equation of the Bohr radius $a_0' = \frac{4\varepsilon'_0 \hbar'^2}{m_e q'^2}$, the nonlinear factors also cancel each other out, yielding:

$$a_0' = a_0 \tag{200}$$

The above results indicate that although the spacetime metric and dielectric parameters undergo nonlinear scaling with the electric potential, the fine-structure constant and the characteristic geometric scale of atoms remain strictly invariant. This ensures that the relative structure of atomic spectra and the microscopic stability of matter are not disrupted by changes in electric potential, proving that the theory is highly self-consistent at the level of quantum mechanics.

3.5 Modified Coulomb's Law

Suppose that in an infinite space, only an ideal point charge q exists. The ideal charge is located at the center of a sphere, and the electric potential on the spherical surface with radius r is ϕ. Let this spherical surface be the equipotential reference frame. Furthermore, at $r = \infty$, $\phi = 0$. According to Electric Potential Relativity, geometric dimensions are independent of the choice and magnitude of the equipotential reference frame. That is, $r' = r$, $\pi' = \pi$, and $\mathrm{d}\boldsymbol{s}' = \mathrm{d}\boldsymbol{s}$. Therefore, $\boldsymbol{E}' = -\frac{\mathrm{d}\phi}{\mathrm{d}r'} = -\frac{\mathrm{d}\phi}{\mathrm{d}r}$, thus:

$$\oiint \boldsymbol{E}' \cdot \mathrm{d}\boldsymbol{s}' = \frac{\mathrm{d}\phi}{\mathrm{d}r}\, 4\pi r^2 \tag{201}$$

From Eq. (201) and the electric field Gauss's law Eq. (147), it is obtained that:

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = -\frac{q}{4\pi\varepsilon_0 r^2}\left(1 - \frac{\phi^2}{\varPhi_0{}^2}\right) \tag{202}$$

Because ϕ is a function of r, let:

$$\frac{\phi}{\varPhi_0} = \mathrm{f}(r) \tag{203}$$

$$\frac{\mathrm{df}(r)}{\mathrm{d}r} = -\frac{q}{4\pi\varepsilon_0 \varPhi_0}\frac{(1-\mathrm{f}(r)^2)}{r^2} \tag{204}$$

Solving this differential equation yields:

$$\frac{\phi}{\varPhi_0} = \tanh\left(\frac{q}{4\pi\varepsilon_0\varPhi_0 r} + \mathrm{c}\right) \tag{205}$$

When $r = \infty$, $\phi = 0$, thus $\mathrm{c} = 0$. Therefore, the electric potential distribution equation for an ideal point charge is:

$$\phi = \varPhi_0 \tanh\left(\frac{q}{4\pi\varepsilon_0\varPhi_0 r}\right) \tag{206}$$

$$\varPhi_0 = \lim_{r\to 0} \varPhi_0 \tanh\left(\frac{q}{4\pi\varepsilon_0\varPhi_0 r}\right) \tag{207}$$

As $r \to 0$, the electric potential ϕ reaches the limit value $\varPhi_0$. That is, the central electric potential of an ideal point charge equals the electric potential limit constant. The electric field distribution equation for an ideal point charge is:

$$E' = \frac{q}{4\pi\varepsilon_0 r^2}\left(1 - \tanh^2\left(\frac{q}{4\pi\varepsilon_0\varPhi_0 r}\right)\right) \tag{208}$$

In nonlinear electromagnetic theory, the value of the charge is independent of the reference frame's electric potential, thus $q' = q$. If two elementary charges are ideal point charges, the Coulomb force between them is F_e'. According to Eq. (208), the modified Coulomb's law is obtained:

$$F_e' = q'E' = \frac{q^2}{4\pi\varepsilon_0 r^2}\left(1 - \tanh^2\left(\frac{q}{4\pi\varepsilon_0\varPhi_0 r}\right)\right) \tag{209}$$

When r is much larger than $\frac{q}{4\pi\varepsilon_0\varPhi_0}$, Eq. (209) reverts to classical Coulomb's law. Taking the limit of Eq. (209) yields:

$$\lim_{r\to 0} F'_{\mathrm{e}} = \lim_{r\to 0} \frac{q^2}{4\pi\varepsilon_0 r^2}\left(1 - \tanh^2\left(\frac{q}{4\pi\varepsilon_0 \Phi_0 r}\right)\right) = 0 \tag{210}$$

Within this nonlinear electromagnetic framework, when two idealized point charges of the same sign approach each other at extremely short distances, the effective Coulomb repulsive force between them no longer diverges to infinity as predicted by classical electrodynamics. Instead, due to the nonlinear geometric corrections of the electric field structure, this repulsive force reaches a maximum at a characteristic "extremal radius," subsequently decays as the distance further decreases, and strictly vanishes at the charge center. Physically, this nonclassical short-distance behavior is profoundly analogous to the phenomenon of "asymptotic freedom" found in the strong interactions of quantum chromodynamics (QCD). This theoretical consequence implies that, under extremely high-potential conditions, same-sign charges could in principle overcome their mutual Coulomb barrier, thereby providing a theoretical basis for extreme physical behaviors—such as direct deep collisions or aggregation—that transcend the permissible limits of classical theory. Ultimately, this offers a novel explanatory mechanism for reconstructing the internal structure and dynamical models of elementary particles.

3.6 Energy of an Ideal Point Charge

In classical electrodynamics, the electric field energy of a point charge inevitably diverges. This problem originates from the way Maxwell's equations treat point charges. If the elementary charge is not a point but is distributed within an extremely small spatial region, it triggers a series of new contradictions. For example, this assumption would destroy the quantized nature of the elementary charge; simultaneously, the repulsive force between partial charges would destroy the stability of the charge. Therefore, most theories in modern physics (such as quantum field theory) still employ the point charge model. However, the point charge energy divergence problem is a long-standing intractable issue that has troubled physicists for over a century.

Physicists have been constantly searching for a fundamental solution to this dilemma. For example, renormalization techniques have alleviated the problem of elementary particle energy divergence to some extent (which involves point charge energy divergence), but this method has also faced sharp criticism from Dirac. It is argued herein that the problem of point charge energy divergence initially stems from classical electrodynamics, indicating that classical electromagnetic theory itself contains flaws [10]. Therefore, only by starting from fundamental physical principles and developing a new electromagnetic theory can this problem be fundamentally resolved.

According to nonlinear electrodynamics, in a reference frame with an electric potential of ϕ, the calculation formula for the electric field energy density per unit volume shares the same mathematical form as classical electrodynamics:

$$w'_{\mathrm{e}} = \frac{1}{2}\varepsilon'_0\, E'^2 \tag{211}$$

Where w'_{e} is the electric field energy density per unit volume, E' is the electric field intensity, and ε'_0 is the vacuum permittivity. Therefore, the electric field energy E_q of an ideal point charge is:

$$E_q = \int_0^{\infty} \frac{1}{2}\varepsilon'_0\, E'^2 dv' \tag{212}$$

E_q is the total electric field energy of the ideal point charge. v' is the volume of the reference frame with electric potential ϕ. According to Electric Potential Relativity, geometric dimensions are independent of the magnitude of the reference frame's electric potential, that is, $\mathrm{d}v' = \mathrm{d}v$. Therefore,

$$dv' = 4\pi r^2 dr \tag{213}$$

Substituting Eqs. (154), (208), and (213) into Eq. (212) yields:

$$E_q = \frac{q^2}{8\pi\varepsilon_0}\int_0^{\infty}\frac{1}{r^2}\left(1-\tanh^2\left(\frac{q}{4\pi\varepsilon_0\Phi_0 r}\right)\right)\mathrm{d}r$$
$$= -\frac{1}{2}q\Phi_0\tanh\left(\frac{q}{4\pi\varepsilon_0\Phi_0 r}\right)\Big|_{r=0}^{r=\infty} = \frac{1}{2}q\Phi_0 \tag{214}$$

This result possesses significant theoretical implications; it demonstrates that starting from the underlying geometric principles of spacetime, the intractable problem of ideal point charge energy divergence, which has troubled classical physics for more than a century, has been naturally eliminated herein. However, it must be pointed out that this work has not yet completely resolved the self-energy divergence problem of elementary particles in realistic physics. This is because real charged elementary particles are not simply geometric point charges; they also possess complex intrinsic attributes such as spin and rest mass. Consequently, the actual energy behavior of elementary particles differs from the ideal point charge model. Although the complete resolution of the elementary particle energy divergence problem still relies on the further development and quantization extension of Electrodynamic Spacetime Theory, the method proposed herein to overcome divergence based on the nature of spacetime undoubtedly opens an entirely new approach to solving this fundamental problem in physics.

3.7 Estimation of the Electric Potential Limit Constant

To provide a necessary order-of-magnitude reference for subsequent experimental verification schemes, a preliminary estimation of Φ_0 is required herein. Deduced from physical mechanisms, Φ_0 might correspond to three key characteristic energy scales: the first is the electroweak scale related to the origin of particle mass [12] (approximately 10^{11}V); the second is the Grand Unified Theory (GUT) scale, predicting the unification of strong, weak, and electromagnetic interactions (approximately 10^{24}V); the third is the Planck scale involving extreme gravitational effects and the limits of spacetime geometry (approximately 10^{28}V). An open attitude is maintained herein regarding the specific value of Φ_0. Φ_0 should be regarded as a new fundamental physical constant. Its precise value ultimately should not rely on theoretical speculation but must be determined by relevant precision experimental observations (such as time dilation, electric potential redshift experiments, or astronomical electric potential redshift observations).

4 Concepts for Experimental Verification and Astronomical Observation

This theory not only resolves the divergence problem of point charges at the microscopic scale but must also remain consistent with existing high-precision electromagnetic experiments at the macroscopic scale. Simultaneously, based on Electrodynamic Spacetime Relativity and nonlinear electromagnetic theory, a series of entirely new predictions for physical effects are proposed herein for experimental verification.

4.1 Compatibility Verification with Classical Gauss's Law Experiments

In classical electrodynamics, the highest-precision experiments used to test the inverse-square law of Coulomb's law are "null result" experiments based on spherical shell geometry (such as the Williams, Faller, & Hill experiment [11]). According to Gauss's theorem, the electric field inside a charged spherical shell should be strictly zero. Any deviation from the inverse-square law would result in a measurable electric field inside the spherical shell; therefore, regarding the nonlinear modification proposed by this theory, it is necessary to verify whether it disrupts this classical conclusion.

In classical electrodynamics, the experimental verification of Gauss's law for the electric field is based on spherical shell geometry: for any point P inside the spherical shell, the area Δa corresponding to the solid angle $\Delta\Omega$ on the spherical shell is proportional to the square of the distance r. The vertically opposite solid angles at point P are equal, $\Delta\Omega_1 = \Delta\Omega_2$, namely:

$$\frac{\triangle a_2}{\triangle a_1} = \frac{r_2^2}{r_1^2} \tag{215}$$

Since the charge density σ on the spherical shell is constant, the charge quantity on the corresponding area is $\triangle q = \sigma \Delta a$; thus:

$$\frac{\triangle q_2}{\triangle q_1} = \frac{\triangle a_2}{\triangle a_1} \tag{216}$$

According to Coulomb's law, the electric field of a point charge is $E \propto \frac{q}{r^2}$. For a small charge $\triangle q$, its contributed electric field component is $\Delta E \propto \frac{\Delta q}{r^2}$. Then:

$$\frac{\triangle E_2}{\triangle E_1} = \frac{\triangle \frac{q_2}{r_2^2}}{\triangle \frac{q_1}{r_1^2}} = 1 \tag{217}$$

In nonlinear electromagnetic theory, although the electric potential Lorentz factor $\gamma(\phi)$ is introduced, for a charged conductive spherical shell, its surface is in an equipotential state ($\phi_1 = \phi_2 = \phi_{\text{shell}}$). Thus, the corresponding nonlinear factor is a constant $\gamma^{-2}(\phi_{\text{shell}})$ everywhere on the surface of the spherical shell. Therefore, according to Eqs. (144) and (217), it follows that:

$$\frac{\triangle E_2'}{\triangle E_1'} = \frac{\gamma^{-2}(\phi_{\text{shell}})\,\triangle E_2}{\gamma^{-2}(\phi_{\text{shell}})\,\triangle E_1} = \frac{\triangle E_2}{\triangle E_1} = 1 \tag{218}$$

This result indicates that the nonlinear factor $\gamma^{-2}(\phi)$, acting as a constant, is completely canceled out in the numerator and denominator. Therefore, the classical Shell Theorem remains strictly valid in this theory, and the total electric field intensity inside the spherical shell is zero. This conclusion provides solid experimental support for the macroscopic self-consistency of the theory.

4.2 Testing the Electric Potential Time Dilation Effect

The preceding derivations indicate that in Electrodynamic Spacetime Relativity, time and space are related to three-dimensional velocity and electric potential. In principle, various experiments can be designed to verify this theory, among which the electric potential time dilation effect might be more easily verified experimentally.

Under completely stationary conditions, when a sufficiently high electrostatic potential difference exists between two reference frames, their time intervals will be found to be different, and the higher the electric potential, the more pronounced this effect becomes. Therefore, an experiment can be designed: after synchronizing two high-precision clocks $\mathrm{T_c}$ and $\mathrm{T_d}$, they are placed respectively into two identical metal spherical shells C and D, which are mutually insulated and relatively stationary. Let chamber C be grounded, setting its electric potential to zero; an ultra-high voltage generator is used to charge the surface of spherical shell D, making spherical shell D charged and maintaining a stable ultra-high electric potential ϕ relative to spherical shell C; after a sufficiently long period, spherical shell D is discharged, rendering its electric potential zero as well.

At this point, bringing the two clocks $\mathrm{T_c}$ and $\mathrm{T_d}$ together to compare their readings will reveal a difference between the proper times τ_{c} and τ_{d} recorded by the respective clocks $\mathrm{T_c}$ and $\mathrm{T_d}$ for their respective reference frames.

$\mathrm{T_c}$ remains constantly in a zero electric potential. In the zero electric potential reference frame, the proper time τ_{c} of $\mathrm{T_c}$ is simply the coordinate time Δt of the zero potential reference frame.

$$\Delta\tau_{\mathrm{c}} = \Delta t \tag{219}$$

$\mathrm{T_d}$ is in a stable electric potential ϕ for the majority of the time, and its proper time τ_{d} satisfies the electric potential time dilation effect Eq. (111).

$$\Delta t = \gamma(\phi)\Delta\tau_{\mathrm{d}} \tag{220}$$

Therefore, from Eqs. (219) and (220), it follows that:

$$\Delta\tau_{\mathrm{d}} = \frac{\Delta\tau_{\mathrm{c}}}{\gamma(\phi)} < \Delta\tau_{\mathrm{c}},\ \ \gamma(\phi) = \frac{1}{\sqrt{1-(|\phi|/\Phi_0)^2}}$$

By comparing the measured values with the theoretically predicted values, the verification results for the theory are obtained.

If the experiment proves the theoretical calculations to be correct, further analysis and calculation of the experimental data will yield the specific value of the limit electric potential Φ_0.

$$\Phi_0 = \frac{\phi}{\sqrt{1-\frac{\Delta\tau_d{}^2}{\Delta\tau_c{}^2}}} \tag{221}$$

Obtaining extremely high electric potentials ϕ in a laboratory is considerably difficult. The advantage of this experimental design is that by extending the experiment's duration and increasing the precision of time measurement, the requirement for the experimental electric potential ϕ can be reduced, thereby enhancing the feasibility of the experiment.

4.3 Potential Michelson Interferometer and Testing the Potential Light Speed Effect

To verify the nonlinear modulation effect of electric potential on the speed of light, an improved Michelson interferometer—the Potential Michelson Interferometer—and related experimental schemes are proposed in this section.

Experimental Principles and Setup: The experiment utilizes a single-mode frequency-stabilized laser as the light source, and the beam is divided into two paths by a beam splitter. The "reference arm" is placed in a zero electric potential environment, while the "measurement arm" passes through the interior of a vacuum metal tube with a high electrostatic potential of ϕ. According to Parameter Comparison Table 1, in nonlinear electromagnetic theory, the spatial scale remains rigid ($\ell' = \ell$), but the flow of time in the high electric potential region dilates relative to external observers. This means that although from the perspective of an observer in the local reference frame, photons still move at a constant C_0 relative to the observer, from the perspective of a laboratory observer, the coordinate speed of light is reduced to $C_0' = \gamma^{-1}(\phi)C_0$.

Fringe Shift Prediction: Suppose the lengths of both arms with a potential difference of ϕ are ℓ, and the laser wavelength is λ_0, then the round-trip flight time t_2 of the beam in the measurement arm will experience a delay compared to t_1 in the reference arm.

$$\Delta t = t_2 - t_1 = \gamma(\phi)\,\frac{2\ell}{C_0} - \frac{2\ell}{C_0} = \frac{2\ell}{C_0}(\gamma(\phi)\ - 1)$$

This time difference will cause a lateral shift of the interference fringes, and the number of fringe shifts *N* is:

$$N = \frac{C_0\Delta t}{\lambda_0} = \frac{2\ell}{\lambda_0}(\gamma(\phi)\ - 1) \tag{222}$$

In this effect, the geometric length of space remains unchanged; the fringe shift originates entirely from the optical path lag caused by relative time dilation within the high electric potential region.

Regarding the construction of the experimental apparatus, this scheme possesses a high degree of flexibility. Initially, a desktop-scale prototype can be designed to verify the principle. For further high-precision experiments, the use of a high-precision Fabry-Pérot Vacuum Cavity is recommended. This technology, through multiple reflections of photons within the vacuum cavity, can fold and amplify the effective optical path of the physical arm length to the kilometer scale or even longer. This significantly enhances the detection sensitivity for nonlinear spacetime effects while ensuring a pure vacuum environment.

4.4 Astronomical Electric Potential Telescope and Electric Potential Redshift Observation

To break through the limitations of electric potential values in ground-based laboratories and meet the needs of exploring the electrical structure of the universe, a novel concept of an "Astronomical Potential Telescope" is proposed herein. Its core idea is to place astronomical telescopes, spectrometers, and other relevant equipment inside a high-potential astronomical observatory (or satellite) with a conductive smooth spherical shell, encapsulating the observation window with movable conductive glass. By providing the highest possible electric potential via an ultra-high-voltage electrostatic generator with adjustable magnitude and polarity, an equipotential ϕ_{obs} space, i.e., a local electric potential reference frame, is formed inside the observatory. As a controllable physical variable, by actively adjusting ϕ_{obs} minute changes in the spectral redshift of charged celestial bodies can be observed, thereby verifying the nonlinear electric potential redshift effect and probing the electric potential structure of celestial bodies in the distant universe.

Traditional astronomical observations can only passively receive photons, unable to eliminate the interference of complex background noise such as cosmological redshift and gravitational redshift. Based on the redshift Eq. (194), the approximate relationship for frequency change after introducing a potential difference in this theory is:

$$\frac{\Delta\nu}{\nu} \approx -\frac{1}{2}\left(\frac{\phi_{\text{obj}}-\phi_{\text{obs}}}{\Phi_0}\right)^2 \ , \ |\ \Delta\phi\ | \ll \Phi_0 \qquad (223)$$

A groundbreaking "Active Potential Modulation" observation scheme is proposed herein. Its core mechanism lies in the fact that the observed total redshift depends not only on the electric potential of the distant celestial body (ϕ_{obj}) but also on the observer's local electric potential (ϕ_{obs}). By placing astronomical telescopes and spectral observation equipment within an equipotential observation cabin, where the electric potential ϕ_{obs} is controllable, and periodically adjusting the observation potential ϕ_{obs} (for example, switching between $+\phi_{\text{obs}}$ and $-\phi_{\text{obs}}$), a time-varying modulation component can be artificially introduced into the incident light signal.

Because the cosmological redshift and gravitational redshift are constant background quantities over short periods, only the "electric potential redshift" component will undergo synchronous changes with variations in ϕ_{obs}. By observing these changes, the unique electric potential effect of this theory can be precisely isolated from the cosmic background, constituting "fingerprint-level" evidence verifying the existence of Electrodynamic Spacetime Relativity. Simultaneously, by observing the symmetric relationship between the variation in ϕ_{obs} and the variation in celestial redshift Δz, the charging characteristics of the target celestial body can be inferred; namely, whether the celestial body is electrically neutral, positively charged, or negatively charged.

The "Astronomical Electric Potential Telescope " transforms the "reference frame electric potential" from a passive theoretical parameter into an active degree of freedom in astronomy. It not only provides an experimental method to verify the existence of Φ_0 on an astronomical scale but also opens an entirely new observational window—namely, "Potential Astronomy"—for probing the charge distribution of compact celestial bodies such as black holes and neutron stars, as well as searching for potential "ultra-high potential celestial bodies" in the universe.

5. Conclusion

In this paper, the electric potential limit constant has been introduced as a hypothesized new fundamental physical constant, and its theoretical consequences for spacetime structure and classical electrodynamics have been systematically explored. On this basis, the formal structure of Special Relativity has been extended from the real domain to the complex and biquaternion domains, leading to a broader mathematical framework termed Electrodynamic Spacetime Relativity (ESR). Within this formulation, standard Special Relativity and Electric Potential Relativity naturally emerge as two limiting sectors of a more general, unified structure.

Based on this geometric framework, a nonlinear electromagnetic theory compatible with Electric Potential Relativity has been developed. Within the assumptions of the present model, the theory is strictly formulated so as to preserve generalized gauge invariance while retaining the absolute invariance of electric charge and the fine-structure constant. The resulting modified Coulomb's law predicts a nonclassical short-distance behavior of the effective interaction. Consequently, for the idealized point-charge model considered in this work, the corresponding electromagnetic field self-energy is naturally regularized to a finite value.

The present framework also predicts macroscopic electric-potential-induced time dilation, redshift, and lensing-like effects, and suggests concrete experimental tests, including the concepts of an electric-potential telescope and an electric-potential Michelson interferometer. In this sense, the theory may provide a profound new perspective for examining the relationship between spacetime geometry and electromagnetic phenomena under extreme potential conditions.

At the same time, the broader implications of this framework for gravitation, quantum-scale physics, and related foundational questions remain open. These issues are not resolved in the present paper and require further rigorous investigation. The main result of this work is therefore not the completion of an ultimate unified theory, but the construction of a self-consistent model framework in which the electric potential limit constant hypothesis successfully leads to a finite electromagnetic field self-energy for the idealized point-charge model, along with a set of associated theoretical consequences that merit further exploration.

6. Outlook

The framework developed in this paper suggests several promising directions for further investigation. If the electric potential limit constant is treated as a hypothesized new fundamental physical constant, then the ESR framework may provide a broader geometric setting in which the relationships among spacetime structure, electrodynamics, and high-potential phenomena can be studied more systematically. At the present stage, however, this framework should still be strictly regarded as a theoretical model whose wider implications remain to be clarified.

One critical direction concerns quantum-scale physics. Because the present formulation introduces a deeper geometric role for electric potential in spacetime structure and employs extensions into the complex and biquaternion domains, it may offer a potential route for exploring whether some features usually associated with quantum descriptions admit a reformulation within a broader continuous spacetime framework. In particular, the nonlinear electromagnetic framework developed in this work, together with the inherent noncommutative algebraic structure of biquaternion spacetime, may provide a mathematical bridge for exploring deep connections between classical field theory and quantum mechanics. Whether such a connection can be established in a mathematically rigorous and physically convincing way remains an open, yet highly compelling, question.

A second direction concerns gravitation and relativistic field theory beyond the present scope. Since General Relativity is built upon the foundational spacetime structure inherited from Special Relativity, any nontrivial extension of the latter must have profound implications for gravitational theory. In this sense, the ESR framework may suggest possible extensions toward gravitation under extreme potential conditions. This leads to a speculative but fascinating "nonlinear electrostatic black hole" model. Since the modified Coulomb's law permits same-sign charges to overcome repulsion and aggregate at extremely short distances, a compact object could maintain an extreme potential approaching $\mathit{\Phi}_0$ deep within its interior—a mechanism that might naturally regularize the infinite energy density singularity from a geometric perspective. Typically, through the accretion of opposite charges, such an object would appear macroscopically neutral to external observers; however, under specific extreme astrophysical mechanisms, it could also exhibit a macroscopic net charge asymmetry. Speculating further, if such objects or large-scale high-potential backgrounds exist universally, the "electric potential redshift" predicted in this framework might offer a novel, non-gravitational contributing mechanism to cosmological redshifts (or certain anomalous astronomical redshifts). Moreover, the introduction of an electric-potential dimension assigns additional electromagnetic structure to the spacetime manifold, suggesting a broader geometric setting in which gravitational and electromagnetic phenomena could potentially be investigated within a common framework. However, no fully coupled gravitational-electromagnetic field theory is established in the present work, and such complex structural issues must be left for future rigorous study.

A third direction concerns empirical and experimental examination. The macroscopic electric-potential-induced effects predicted in this paper, including time dilation, redshift, and lensing-like behavior, suggest possible targets for observational and laboratory tests. Proposed concepts such as an electric-potential telescope, an electric-potential Michelson interferometer, and related high-voltage equipotential systems may serve as realistic starting points for assessing whether the present framework yields physically measurable consequences.

More broadly, the ultimate significance of the present work lies not in claiming that these monumental foundational questions have already been resolved, but in proposing a structured theoretical framework in which they may be posed in an entirely new way. Whether the electric potential limit constant hypothesis can ultimately contribute to a deeper foundational reformulation of classical and modern physics remains to be determined by further mathematical development, physical analysis, and strict experimental scrutiny.

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